

TECHNICAL REPORT

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NEW TABLES FOR AIR FLOW

by

Harold J. Hoge

March 1970

UNITED STATES ARMY
NATICK LABORATORIES
Natick, Massachusetts 01760



Pioneering Research Laboratory

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Project Reference:
1T061102B11A-07

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Foreword

A number of tables giving the properties of air in 1-dimensional flow have been published in the past. When a new set of tables such as the present one is issued, the question is therefore: "What desirable property do these tables have that distinguishes them from previous tables?" The answer is that the present tables are more general, and expedite the solution of many problems for which conventional tables are not suited. The use of the Mach number as a variable results in tables that are independent of the stagnation temperature of the gas. The present tables go a step farther. A new force variable is introduced in such a way as to make the tables independent of stagnation pressure, while retaining the usual independence of stagnation temperature.

The project reference for this work is 1T061102B11A-07.

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ABSTRACT

New tables for the compressible flow of air have been prepared, based on 1-dimensional flow with $k = 1.4$. A new dimensionless force variable $\phi = kPA/c_*\dot{m}$ is introduced and it is shown that this variable is independent of the stagnation pressure of the flow. This feature makes the new tables applicable to flows of different stagnation temperatures and pressures in much the same way that conventional tables are applicable to flows of different stagnation temperatures only. The argument of the new tables is the force-variable ϕ ; the functions tabulated are λ (M^*), M , j , T/T_0 , c/c_* , P/P_0 , ρ/ρ_0 , A/A_* , $\ln(A/A_*)$, and $4fL_*/D$. The tables are divided into white regions, in which linear interpolation introduces no significant error; and gray regions, in which linear interpolation can introduce error.

NEW TABLES FOR AIR FLOW

1. Introduction

New tables containing the information needed for solving the usual problems of 1-dimensional, compressible flow of air have been constructed. Several advantages are claimed for them. They are more compact than previous tables yet they permit the solution of most of the problems that can be solved with more extensive tables, including problems of general flow, isentropic flow, Fanno-line flow, Rayleigh-line flow, the determination of friction coefficients, and the calculation of properties across normal shocks. Many fluid-flow problems can be solved more directly and easily with the present tables than with previous tables, as will be demonstrated later by examples. Other problems can be solved with equal ease with the present tables and with previous tables.

The most important new feature of the present tables is the use of a new force-variable ϕ . The variable ϕ has been chosen because it is easily computed from the commonly measured quantities and because its use greatly increases the generality of the tables without increasing their complexity. Just as a table employing M rather than velocity can be used for a flow of any stagnation temperature, a table employing ϕ rather than pressure and area can be used for a flow of any stagnation pressure, as will be demonstrated later.

As velocity variables, both the Mach number M and the velocity-number λ are given in the present tables. The velocity number $\lambda \equiv u/c_*$, where u is the stream velocity and c_* is the velocity that the stream would reach at a sonic throat. The variable λ is often represented by M^* . In the present report λ is generally preferred to M because it leads to simpler expressions for most of the quantities to be calculated; M is included for those who prefer it and for use in those problems where it is more convenient than λ .

The tables of this report consist of three small auxiliary tables and a main table. The latter (Table 4) contains all the calculated functions. The functions are divided into 4 groups: general, isoenergetic, isentropic, and frictional. Each group is sometimes referred to as a table or tables although all are printed side by side as parts of Table 4. Because ϕ is usually the first of the tabulated quantities to be found experimentally, it has been chosen as the argument of the table.

The first four columns of Table 4 constitute the general table. Most of the new and useful features of the present tables are associated with this table. Without going into details here, it will be mentioned that this table is used when enough properties are known to define a state and other properties of the state are desired, and that it is also used for other purposes by imposing various conditions on it. Thus, when certain conditions are imposed on ϕ , the general table becomes a Fanno-line table.

Columns 5 and 6 in Table 4 constitute the isoenergetic table. The reference states in this table are the stagnation state (subscript o) and the sonic state (subscript*). In this table the reference states are isoenergetic with the general state under consideration. Stated otherwise, the isoenergetic tables apply in situations where the stagnation temperature T_o does not change.

Columns 7 to 10 of Table 4 constitute the isentropic table. The reference states in this table are the stagnation state and the sonic state having the same entropy as the general state under consideration. Note that in most applications of the isentropic tables the flow is both isentropic and isoenergetic. If M is used rather than ϕ , the present isentropic tables are entirely conventional.

Column 11, the final function in Table 4, constitutes the frictional table; it is the conventional friction function.

The grouping of the tables in Table 4, so that the conditions of applicability of each group of functions can be seen at a glance, is considered important. In previous tables, general relations have often been given in "isentropic" tables. Users, especially occasional users, have often been unaware that these relations were general. As a result many laborious and unnecessary computations have probably been made by hand when the tables could have been used.

Portions of Table 4 are shaded. In the shaded portions, linear interpolation can result in reduced accuracy. In the unshaded portions, linear interpolation will introduce no significant error. Methods whereby the accuracy of the unshaded portions can also be achieved in the shaded portions are presented. These methods include higher-order interpolation and also the use of the original and modified equations.

Tables for the compressible flow of gases have been published by Keenan and Kaye [1], Emmons [2], the British Compressible-Flow Tables Panel [3], the Ames (NACA) Research Staff [4], Bauer and Marek [5], and

many others. For air the isentropic exponent k ($= c_p/c_v$) is usually taken as 1.4, and this value has been accepted in the present report. The present tables are for air only. Conventional tables for other values of k are given by Keenan and Kaye ($k = 1.0, 1.1, 1.2, 1.3, 1.4$, and 1.67) and by Bauer and Marek. The latter give tables at intervals of 0.01 for $k = 1.01$ to $k = 1.41$. The tables in the present report are intended for the same general purposes as those referred to above.

2. Fluid-Flow Equations in Terms of ϕ and λ

Since the variables used in the present tables are not the conventional ones, a number of the more important fluid-flow equations will be derived in terms of the accepted variables, starting from conventional forms of the equations of continuity, energy, and momentum, and using other well-known relations. Sections 2, 3, and 4 of this report give the theoretical basis of the present tables and other general information. These sections may be omitted by readers whose goal is to learn how to solve a particular problem. The tables, instructions for their use, and worked examples are given in sections 5, 6, and 7 of this report.

The new force variable ϕ will first be defined; it is

$$\phi \equiv \frac{kPA}{c_* \dot{m}} \quad (1)$$

where P is the pressure, A is the cross-section area of the flow, \dot{m} is the mass rate of flow, and the quantities k and c_* have been previously defined. The force is of course PA . The normalizing factor $k/c_* \dot{m}$ makes ϕ dimensionless and gives it the value 1 for sonic flow. It will be shown later that under the usual conditions of flow, ϕ is a function only of λ (or of M). If k , c_* , \dot{m} , and λ are held constant and A is varied, ϕ does not change. This is true because as A increases P decreases, and vice versa, in such a way that the product PA remains constant. Stated otherwise, ϕ is independent of the stagnation pressure P_0 of the flow. The very useful fact that the PA -product is independent of stagnation pressure has not been widely or clearly recognized in the past. Once it is recognized, the advantages of using a variable such as ϕ become obvious.

The constancy of the PA -product is made plausible by recalling some of the concepts of kinetic theory. According to kinetic theory, static pressure is proportional to the rate at which momentum is carried in either direction across a unit plane fixed relative to the gas, by the thermal agitation of the molecules.

It turns out that if we change the cross-section area of a flow, keeping \dot{m} and λ (and hence also T) constant we do not change the number of molecules carrying thermal momentum or the rate at which they carry it. In macroscopic terms, when A is doubled P is cut in half and the PA product does not change.

Equation of Continuity. The equation of continuity is

$$\dot{m} = \rho u A \quad (2)$$

where ρ is the density of the flow. Using the ideal gas law $P = \rho RT$ to eliminate ρ , the equation of continuity becomes

$$\frac{PA}{\dot{m}} = \frac{RT}{u} \quad (3)$$

We multiply this equation by k/c_* , write T as $T_*(T/T_*)$ and u as $c_* \lambda$ to obtain

$$\frac{kPA}{c_* \dot{m}} = \frac{kRT_*}{c_*^2} \frac{1}{\lambda} \frac{T}{T_*} \quad (4)$$

The subscript star indicates as usual a sonic reference state. In view of the fact that

$$c = \sqrt{kRT} \quad (5)$$

we see that kRT_*/c_*^2 is equal to unity.

Then, using eq (1), eq (4) reduces to

$$\phi = \frac{1}{\lambda} \frac{T}{T_*} \quad (6)$$

Using the relation

$$\frac{T_*}{T_0} = \frac{2}{k+1}, \quad (7)$$

where T_0 is the stagnation temperature of the flow, eq (6) may also be written

$$\phi = \frac{k+1}{2} \frac{1}{\lambda} \frac{T}{T_0} \quad (8)$$

This is the desired form of the equation of continuity.

Combined Equations of Continuity and Energy. The energy equation will be accepted in the form

$$\frac{T}{T_0} = 1 - \frac{k-1}{k+1} \lambda^2; \quad (9)$$

this form is of course valid only when c_p , the specific heat at constant pressure, is constant. When T/T_0 is eliminated between eqs (8) and (9) we obtain

$$\phi = \frac{k+1}{2} \frac{1}{\lambda} - \frac{k-1}{2} \lambda, \quad (10)$$

which is the desired form of the combined equations of continuity and energy.

We can now prove the statement made earlier regarding the constancy of the PA -product of a stream. It is clear from eq (10) that ϕ is a function of λ only, with no restrictions on P_0 , T_0 , or \dot{m} . To examine the situation more closely we will rewrite eq (10), restoring the value of ϕ given in eq (1)

$$PA = \frac{c_* \dot{m}}{k} \left(\frac{k+1}{2} \frac{1}{\lambda} - \frac{k-1}{2} \lambda \right). \quad (11)$$

This equation shows that when the quantities on the right-hand side of the equation remain constant, P and A can vary only in such a way that their product remains constant.

The special case in which eq (11) is applied to sonic flow ($\lambda = 1$) is of interest. In this case we find

$$\frac{P_* A_*}{c_*} = \frac{\dot{m}}{k}. \quad (12)$$

The quantity $P_* A_* / c_*$ is evidently an invariant for any 1-dimensional flow unless there is addition or loss of mass, or a process such as dissociation that changes k . We may change P_0 , T_0 , or A in a flow in any physically possible way, changing P_* , A_* , and c_* separately, but the quantity $P_* A_* / c_*$ will not change. In view of eq (12), $k/c_* \dot{m} = 1/P_* A_*$ and

$$\phi = \frac{PA}{P_* A_*}. \quad (13)$$

This could have been adopted as the definition of ϕ instead of eq (1). The definition involving P_*A_* is perhaps more elegant mathematically, but the other is preferred for two reasons: first, eq (1) is more convenient in practice because it contains the experimental quantities from which ϕ is usually calculated; second, eq (1) avoids any confusion that might arise concerning the meaning of P_* and A_* . These quantities can have more than one meaning, depending on whether an isentrope, a Fanno line, or a Rayleigh line is being discussed.

Impulse Function. The impulse equation, also known as the momentum equation, is

$$J = PA + \dot{m}u. \quad (14)$$

For convenience we will refer to J as the impulse of the stream. Note that J has the dimensions of force, whereas impulse is conventionally the product of force and time. We multiply eq (14) by the same factor previously used in the definition of ϕ , obtaining

$$\frac{kJ}{c_*\dot{m}} = \frac{kPA}{c_*\dot{m}} + \frac{k\dot{m}u}{c_*\dot{m}}. \quad (15)$$

We define a normalized impulse function

$$j \equiv \frac{kJ}{c_*\dot{m}} \quad (16)$$

so that eq (15) reduces to

$$j = \phi + k\lambda. \quad (17)$$

Eliminating ϕ between eqs (10) and (17) gives

$$j = \frac{k+1}{2} \left(\lambda + \frac{1}{\lambda} \right). \quad (18)$$

This is the desired form of the impulse equation. It is particularly useful because it gives j as a function of λ only, with no reference to P or A or the product PA , and with no reference to T except by way of c_* , which occurs in λ . In the derivation of eq (18) the equations of continuity, energy, and momentum were all used. Any solution of eq (18) is able to satisfy all three of these equations, but if information about pressures and areas is required, it is necessary to use some other equation in addition to eq (18). For

example we may know λ , get j from eq (18); then go to eq (17) to find ϕ ; finally from ϕ we get P or whatever ultimate quantity is needed.

The equations derived above include the most important equations of 1-dimensional fluid flow. There are a great many more that could be derived, but from this point on the treatment will be confined mainly to those additional equations that are actually needed in connection with the present tables.

Equations (10) and (18) give ϕ and j respectively as functions of λ . The inverse equations are often needed. Solving eqs (10) and (18) for λ we obtain

$$\lambda = -\frac{\phi}{k-1} + \left[\left(\frac{\phi}{k-1} \right)^2 + \frac{k+1}{k-1} \right]^{\frac{1}{2}} \quad (19)$$

and

$$\lambda = \frac{j}{k+1} + \left[\left(\frac{j}{k+1} \right)^2 - 1 \right]^{\frac{1}{2}} \quad (20)$$

Only the positive sign of the radical is retained in eq (19) because λ is always positive. Both signs are retained in eq (20); the + sign corresponds to supersonic flow and the - sign to subsonic flow. The two conjugate values corresponding to the same value of j satisfy the relation $\lambda_1 \lambda_2 = 1$, and correspond to the velocity numbers on the two sides of a normal shock. The minimum value of j is $k+1$; lower values of j are forbidden.

Relations Between c , c_* , M , and λ . From eq (5) we obtain

$$\frac{c}{c_*} = \sqrt{\frac{T}{T_*}}; \quad (21)$$

in view of eq (6) there is an alternative expression

$$\frac{c}{c_*} = \sqrt{\phi \lambda}. \quad (22)$$

The relation between the Mach number M and the velocity number λ can be conveniently obtained in terms of c/c_* . Since $M = u/c$ and $\lambda = u/c_*$

$$M = \frac{\lambda}{c/c_*}. \quad (23)$$

From eqs (22) and (23) a very simple relation can be found;

$$M^2 = \frac{\lambda}{\phi} . \quad (24)$$

One of the desirable properties of a mathematical analysis is that it yield simple, compact equations. Looking at, for example, eqs (6), (10), (18), and (22) from this point of view we note that they are indeed simple and compact. The choice of ϕ and λ as variables has led to simple, convenient equations based on simple concepts. It is possible that some other choice of variables would be still better but this does not seem very likely.

Isentropic Relations. The isentropic relation between pressure and temperature is

$$\frac{P}{P_0} = \left(\frac{T}{T_0} \right)^{\frac{k}{k-1}} \quad (25)$$

the values of T/T_0 being obtained from eq (9). Similar relations based on isentropic temperature ratios hold for other quantities. Thus

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0} \right)^{\frac{1}{k-1}} \quad (26)$$

and

$$\frac{A}{A_*} = \frac{1}{\lambda} \left(\frac{T}{T_*} \right)^{\frac{1}{k-1}} . \quad (27)$$

However, when P/P_0 has been found from eq (25) it is more convenient to obtain A/A_* from eq (13) than from eq (27). Expressing P in terms of P/P_0

$$\frac{A}{A_*} = \frac{P_*}{P_0} \frac{\phi}{P/P_0} . \quad (28)$$

Also, when A/A_* has been found it is easier to get ρ/ρ_0 from the equation of continuity than from eq (26). Writing eq (2) for a general state and for the corresponding isoenergetic sonic state we find

$$\frac{\rho}{\rho_0} = \frac{\rho_*/\rho_0}{\lambda (A/A_*)} . \quad (29)$$

Stagnation Pressure. The change in stagnation pressure of a stream is a useful quantity closely related to the entropy change. Changes are usually expressed as ratios. When values of P are known for two states of a flow the most straightforward way to get the stagnation-pressure ratio is to calculate ϕ , get P/P_o from ϕ , and then use the equation

$$\frac{P_{o2}}{P_{o1}} = \frac{P_2}{P_1} \frac{(P/P_o)_1}{(P/P_o)_2} \quad (30)$$

However, there is another way of getting P_{o2}/P_{o1} that is sometimes more convenient.

The alternative to eq (30) involves A/A_* instead of P/P_o , and the greater convenience of the alternative comes from the fact that in most problems areas remain constant, whereas pressures change. Applying eq (12) to any two states of an isoenergetic flow ($c_* = \text{constant}$), we see that

$$(P_* A_*)_1 = (P_* A_*)_2 \quad (31)$$

We solve this equation for P_{*2}/P_{*1} and use the fact that $P_{*2}/P_{*1} = P_{o2}/P_{o1}$, obtaining

$$\frac{P_{o2}}{P_{o1}} = \frac{A_{*1}}{A_{*2}} \quad (32)$$

or

$$\frac{P_{o2}}{P_{o1}} = \frac{A_1}{A_2} \frac{(A/A_*)_2}{(A/A_*)_1} \quad (33)$$

If A_1/A_2 remains constant during a series of calculations and P_2/P_1 does not, eq (33) is preferable to eq (30). Note that for constant area flow A_1/A_2 drops out of the equation.

Entropy and Efficiency. The entropy s of a gas relative to the entropy s_r in some reference state is

$$s - s_r = c_p \ln \frac{T}{T_r} - R \ln \frac{P}{P_r} \quad (34)$$

This equation is valid both for flowing gases and for gases at rest. If we put s equal to a constant, s_0 , eq (34) becomes the equation of an isentrope. The point of greatest interest on an isentrope is the stagnation state; its temperature is T_0 and its pressure P_0 . Thus

$$s_0 - s_r = c_p \ln \frac{T_0}{T_r} - R \ln \frac{P_0}{P_r} \quad (35)$$

If we use this equation to calculate the change in entropy between two isoenergetic stagnation states ($T_{01} = T_{02}$) we find

$$\frac{s_{02} - s_{01}}{R} = - \ln \frac{P_{02}}{P_{01}} \quad (36)$$

This quantity can be taken directly from Table 3.

Friction Function. The friction factor f and the friction function $4fL_*/D$ are useful in problems of 1-dimensional flow if f is constant or does not change rapidly. Actual flows are never completely 1-dimensional and f is not usually constant, but the treatment based on these assumptions forms a convenient starting point for more accurate treatments and is often a good approximation itself. The friction factor is

$$f = \frac{\tau}{\frac{1}{2} \rho u^2} \quad (37)$$

where τ is the shearing stress at the wall. The force of friction exerted on a stream in a tube of diameter D and length dl is

$$dF = -\pi D dl \tau; \quad (38)$$

using eq (37)

$$dF = -\frac{1}{2} \pi D f \rho u^2 dl \quad (39)$$

In a constant-area channel the change in impulse dJ is equal to the force of friction dF . But from eqs (16) and (18)

$$dJ = c_* \dot{m} \frac{k+1}{2k} \left(1 - \frac{1}{\lambda^2} \right) d\lambda \quad (40)$$

for an isoenergetic flow ($c_* = \text{constant}$).

We equate dF and dJ and eliminate \dot{m} with eq (2), obtaining

$$-2 \frac{f}{D} d\ell = \frac{k+1}{2k} \left(\frac{1}{\lambda} - \frac{1}{\lambda^3} \right) d\lambda. \quad (41)$$

We will adopt the usual artifice of integrating this equation along a Fanno-line path, from a general state to the corresponding sonic state, assuming a hypothetical channel length just sufficient to bring the flow to the sonic state. Assuming f constant, the integral is

$$-\left[2 \frac{f}{D} \ell \right]_{\ell}^{\ell_*} = \frac{k+1}{2k} \left[\ln \lambda + \frac{1}{\lambda^2} \right]_{\lambda}^1. \quad (42)$$

The distance from the general state at ℓ to the sonic state at ℓ_* will be represented by L_* . Evaluating eq (42) and multiplying by -2

$$\frac{4fL_*}{D} = \frac{k+1}{2k} \left[2 \ln \lambda + \frac{1}{\lambda^2} - 1 \right] \quad (43)$$

which is the friction equation.

Earlier in this report it was stated that the use of λ usually led to simpler expressions than the use of M . In support of this statement eq (43) will be given in terms of M ; it is

$$\frac{4fL_*}{D} = \frac{1-M^2}{kM^2} + \frac{k+1}{2k} \ln \frac{(k+1) M^2}{2(1 + \frac{k-1}{2} M^2)}; \quad (44)$$

eq (43) is considerably simpler than eq (44).

Modified Forms of Some Equations. In some of the gray regions of Table 4, especially those where either ϕ or the function approaches ∞ , interpolation becomes inconvenient and even impossible. A gray-area problem that occurs quite often is to find λ from ϕ , when ϕ is large and λ is small. For this situation it is convenient to write eq (10) in the form

$$\lambda = \frac{\frac{k+1}{2}}{\phi + \frac{k-1}{2} \lambda}. \quad (45)$$

This equation must be solved by successive approximations since the unknown, λ , appears on both sides. However, when ϕ is large and λ is small it converges very rapidly; it is then easier to solve than eq (19). For the first approximation the value of λ on the right-hand side may be taken as zero, or the nearest tabular entry in Table 4 may be used.

Another way of expediting the calculation of accurate values in the gray areas of Table 4 is to use the early terms of series expansions of the exact expressions. As an example, consider the finding of P/P_0 when ϕ is large and λ is small. The exact expression for P/P_0 is eq (25), which is somewhat inconvenient to evaluate. We eliminate T/T_0 from eq (25) with eq (9) and expand the result by the binomial theorem to obtain

$$\frac{P}{P_0} = 1 - \frac{k}{k+1} \lambda^2 + \frac{1}{2} \frac{k}{k+1} \frac{1}{k+1} \lambda^4 - + \dots \quad (46)$$

This equation is normally used only in the region where the terms in λ^4 and above can be neglected.

The basic equations in terms of ϕ and λ needed in the calculation and the use of the tables have now been derived. A few additional equations are given without derivation in Section 5 of this report. Among them are some series expansions of other functions similar to eq (46).

3. Calculation of the Tables

The present section describes the methods used for calculating each function given in the present report. The results are presented in Section 6 as figures (pages 26-28) and as tables (pages 29-40). Figures 1 and 2 show the various quantities as functions of ϕ . Figure 3 gives the same information in the more familiar form in which M rather than ϕ is the abscissa.

Table 1 contains the values of R used in the present report and also some often-used conversion factors. Table 2 is for use in finding c_* from T_0 and vice versa. Writing eq (5) for sonic flow and eliminating T_* with eq (7)

$$c_* = \sqrt{\frac{2k}{k+1} RT_0} \quad (47)$$

$$= c_1 \sqrt{T_0} \quad (48)$$

Table 2 gives values of the constant $C_1 = \sqrt{2kR/(k+1)}$. The need for several entries in the table comes from the fact that conversion factors have been included in C_1 , to give c_* in any one of three sets of units, when T is given either in $^{\circ}\text{R}$ or $^{\circ}\text{K}$.

Table 3 is for use with eq (1). For actual computations, it is convenient to write this equation in the form

$$\phi = C_2 \frac{\text{kPA}}{c_{*m}} \quad (49)$$

where C_2 is the conversion factor required to reconcile the units of PA with those of c_{*m} , making ϕ dimensionless. Table 3 was calculated with the aid of the conversion-factor handbook of Zimmerman and Lavine [6]. Later the table was verified by use of the table of fundamental constants and conversion factors given in reference [7]. No discrepancies were found.

In calculating Table 4, the force-variable ϕ was taken as the argument; λ was calculated from ϕ using eq (19); j was then found from λ using eq (18). Values of T/T_0 were calculated from eq (9), after which c/c_* was found from

$$\frac{c}{c_*} = \sqrt{\frac{T_0}{T_*}} \sqrt{\frac{T}{T_0}} \quad (50)$$

which is a modified form of eq (21). Next M was calculated from eq (23).

The isentropic functions of Table 4 were calculated from the previously-found values of T/T_0 . First P/P_0 was found from eq (25); next A/A_* was found from eq (28); finally ρ/ρ_0 was found from eq (29). The friction function was calculated from eq (43).

The length of Table 4 represents a compromise between convenience and compactness. The entries are spaced closely enough so that there are large regions in which linear interpolation introduces no significant error. As mentioned earlier, the white areas of the table permit linear interpolation without loss of accuracy, whereas in the areas with a gray shading some loss of accuracy can result. Where full accuracy is required in the gray regions, various alternatives are available. In some cases the original equations from which the tables were calculated are convenient to use. In others a modified form of an equation, or a series expansion, may be useful. But in the majority of cases it will be preferable to resort to higher-order interpolation. Unfortunately there are regions where interpolation of any order becomes impractical. These are the regions, generally at the ends of the

table, where either the argument or the function approaches infinity. In these regions some form of the defining equations should be used.

For convenience, the formulas for linear and 4-point (cubic) interpolation without the use of differences will be given. These are the formulas that are most convenient when a desk calculator is used. For linear interpolation between the two tabulated values f_0 and f_1 (do not confuse these f 's with the friction factor) the formula is

$$f = (1-p)f_0 + pf_1 \quad (51)$$

where p is the fraction of the tabular interval at which f is desired. Note that the sum of the multipliers $1-p$ and p is unity. Cumulative multiplication should be used, and if pf_1 is calculated first the quantity $1-p$ may be obtained as an auxiliary quantity (the counting dials are run backward).

When higher-order interpolation is used, 4-point is often used rather than 3-point (quadratic), because 4-point interpolation is more accurate and little, if any, more difficult. The recommended equation is

$$f = A_{-1}f_{-1} + A_0f_0 + A_1f_1 + A_2f_2 \quad (52)$$

Except at the ends of the table where f_{-1} or f_2 may not exist, the four tabular values are chosen so that f lies between f_0 and f_1 . The A 's are interpolation coefficients that are functions of p , the fraction of the tabular interval. Reference [8] contains values of the A 's tabulated at intervals of 0.0001 in p .

As a check against computational errors, the first and second differences of all functions in Table 4 were machine-computed and printed as part of the basic computer program. The differences, especially Δ^2 , were examined carefully for errors. Eighteen significant digits were carried in all of the machine computations. Before printing out, the functions were all rounded to 4 decimal places and the differences generally to 5 decimal places. All rounding was to the nearer figure. As an additional check for errors, a number of spot comparisons were made between the present tables and those of Keenan and Kaye [1] and the British Compressible Flow Tables Panel [3]. The functions T/T_0 , P/P_0 , f/f_0 , and A/A_0 , which appear in all 3 sets of tables, and the functions λ and $4fL^*/D$, which appear in reference [1] and in the present tables, were compared at $M = 0, 0.1, 0.2 \dots 2.0, 2.5, 3.0, 4, 5, 10$, and ∞ .

Additional comparisons between reference [1] and the present tables (at the values of M given above) were made by using the relation $j = (k+1) (F/F_*)$, and by using the fact that, along a Fanno line, P/P_* is numerically equal to ϕ . In reference [1], ϕ was obtained both from the Fanno-line table (Table 42) and from the isentropic table (Table 30). For the latter, the relation $\phi = (P_0/P_*) (PA/P_0 A_*)$ was used.

The discrepancies shown up by the spot comparisons were essentially those to be expected from rounding-off and interpolation errors. In the case of a few functions there were slight indications of consistent discrepancies, but these were on the border line of detectability. The accuracy of each of the 3 tables compared appears to be adequate for all ordinary uses.

The second differences of the various functions were used as the basis for dividing Table 4 into white and gray areas. According to interpolation theory the maximum difference between the results of linear interpolation and quadratic interpolation reaches $1/2$ unit in the last decimal place of a table when the second difference equals 4 in that decimal place. In the white areas of Table 4 the second differences are less than 4 and in the gray areas they are normally 4 or greater.

4. Discussion

The principal advantage claimed for the present tables as compared with previous tables is their greater generality. Whereas the use of the Mach number or velocity number makes a single table valid for flows of different stagnation temperatures, the use of the force-variable ϕ also makes the same table valid for flows of different stagnation pressures. The generality comes from the way in which λ (or M) and ϕ are defined. The tables are independent of mass rate of flow because of the presence of \dot{m} in the normalizing factor $k/c_* \dot{m}$; they are independent of stagnation temperature because of the presence of c_* in the normalizing factor and in the definition of λ ; and they are independent of stagnation pressure because the factor $k/c_* \dot{m}$ is independent of stagnation pressure.

Previous tables (excluding shock tables) have usually consisted of a single "isentropic" table or of an isentropic table, a Fanno-line table, and a Rayleigh-line table. The present tables consist of a general table, an isoenergetic table, an isentropic table, and a frictional table.

The general table contains the functional relationships that hold for any 1-dimensional flow. Since λ (or M) is uniquely determined by ϕ , the general table furnishes a convenient means of finding λ (or M) from measured quantities. Thus if we know A , T_o , and \dot{m} we can calculate c_* from T_o and find ϕ ; then we enter the table to find λ . All other tabulated properties of the flow are also immediately available.

When the conditions of constant T_o (constant c_*) and constant A are imposed on ϕ , the general table becomes equivalent to a Fanno-line table. The variable ϕ , though defined in an entirely independent way, is numerically equal to the pressure ratio P/P_* given in a Fanno-line table.

In Rayleigh-line flow the impulse J and the area of the flow remain constant, but T_o (and c_*) change. Thus according to eq (16) the quantity $c_* J$ remains constant on a Rayleigh line. By imposing the conditions of constant A and constant $c_* J$ on the general table it may be used as a Rayleigh-line table. The conditions are not so easy to work with as those involved in Fanno-line flow; however, they permit Rayleigh-line problems to be solved with the general table. In solving problems other than Fanno-line and Rayleigh-line problems it may be desirable to impose still other conditions on the general table. For example we may impose the condition of constant temperature or of constant pressure.

The isoenergetic functions in Table 4 are separated from the isentropic functions to emphasize the fact that an isoenergetic flow need not be isentropic and usually is not. It is also true that an isentropic flow need not be isoenergetic, but it usually is. The principal exception occurs when mechanical work is done by or on the gas. Thus in most of the problems to which the isentropic functions are applied the flow is both isoenergetic and isentropic and both T_o and P_o are constant.

The quantity $PA/P_o A_*$ tabulated by various authors is closely related to ϕ of the present report; we see from eq (13) that

$$\frac{PA}{P_o A_*} = \frac{P_*}{P_o} \phi. \quad (53)$$

The quantity P_o appearing in this equation must be identified as the local stagnation pressure. If P_o is taken to be the stagnation pressure in some initial upstream state, and the flow is not isentropic, eq (53) will not hold.

The quantity $PA/P_o A_*$ has most of the desirable properties of ϕ since it is equal to a constant (P_*/P_o) times ϕ . The relations between $PA/P_o A_*$, M , λ , and F/F_* given by Keenan and Kaye [1] and by others are general when P_o is interpreted as the local stagnation pressure, although they are presented as parts of isentropic tables. For example the relation between M and $PA/P_o A_*$ tabulated by Keenan and Kaye may be used to calculate M from the pressure and the flow per unit area, since eq (53) may be written in the form

$$\frac{PA}{P_o A_*} = \frac{P_*}{P_o} \frac{kPA}{c_* m} \quad (54)$$

After $PA/P_o A_*$ has been calculated from this equation the table is entered and M found directly. This is more convenient than using the mass flow parameter as described by Shapiro (reference [9], p82). The use of the general section of the present tables is somewhat more convenient than either of these methods.

In this report ϕ has been used in preference to $PA/P_o A_*$ because of its greater convenience and simplicity, and because it has the value 1 for sonic flow rather than the value P_*/P_o . Note however that, when ϕ is less than 1, λ and M are greater than 1, and vice versa.

Since a high degree of generality is claimed for the present tables, mention should be made of the generalized compressible-flow function of Benedict and Steltz [10]. This function is

$$\Gamma = \frac{\left(\frac{P}{P_o}\right)^{\frac{1}{k-1}}}{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}} \left[\frac{1 - \left(\frac{P}{P_o}\right)^{\frac{k}{k-1}}}{\frac{k-1}{k+1}} \right]^{\frac{1}{2}} \quad (55)$$

the authors show that it satisfies the equation

$$\left(\frac{T_{O1}}{T_{O2}}\right)^{\frac{1}{2}} \frac{P_{O2}}{P_{O1}} \frac{A_2}{A_1} = \frac{\Gamma_1}{\Gamma_2} \quad (56)$$

With eq (56) the generality of the present treatment is achieved in a somewhat roundabout way. It can easily be shown that Γ is not really a new function at all, that in fact

$$\Gamma = \frac{A_*}{A} \quad (57)$$

where A_*/A is the isentropic area-relation.

The rather lengthy treatment of critical states and of entropies given by Benedict and Steltz is unnecessary, as was pointed out by A. J. W. Smith in his discussion of the paper (reference [10], p 68). Smith derived a function X, previously used by others in England, and showed that Γ was equal to a constant times X. It is not clear whether any of these workers were aware that their functions were simply variations of A_*/A . Probably Benedict and Steltz considered Γ as a function that in some situations was equal to A_*/A but in other situations had a different meaning. At any rate, they did not emphasize the relationship, nor point out to the reader that the reciprocal of the function Γ had already been given in existing tables.

It will be shown that eq (56) actually introduces the independence of stagnation pressures that is achieved in the present treatment by the use of ϕ . We first eliminate Γ_1 and Γ_2 from eq (56) by using eq (57), obtaining

$$\left(\frac{T_{O1}}{T_{O2}}\right)^{\frac{1}{2}} \frac{P_{O2}}{P_{O1}} \frac{A_{*2}}{A_{*1}} = 1. \quad (58)$$

The first term in this equation may be written as c_{*1}/c_{*2} , and the second as P_{*2}/P_{*1} , giving

$$\frac{c_{*1}}{c_{*2}} \frac{P_{*2}}{P_{*1}} \frac{A_{*2}}{A_{*1}} = 1$$

or

$$\left(\frac{P_* A_*}{c_*}\right)_2 = \left(\frac{P_* A_*}{c_*}\right)_1 \quad (59)$$

Since this relation holds between any two points in a flow, the quantity $P_* A_* / c_*$ must be a constant for the flow. We do not get the value of this constant from the equation of Benedict and Steltz, but from eq (12) we see that it is \dot{m}/k . Since P/P_* and A/A_* are known to be functions of λ (or M) only, PA/c must likewise be a function of λ only, and the generality of eq (56)* has been demonstrated.

Approximate solutions to difficult problems can often be obtained by assuming the flow to be isentropic. In other cases the solution to a problem can be obtained for the isentropic case, but the result actually is not limited to isentropic flow. Benedict and Steltz derived eq (56), which contains a number of isentropic relations, but, as they point out, their result is not limited to isentropic flow. Other authors have also developed "isentropic" functions that are not limited to isentropic flow. When practical it is desirable to derive general relations first, and limit them to special cases only when necessary. This has been done in the present treatment.

5. Instructions and Working Equations

This section of the present report contains the rules to be followed and the equations to be used in solving problems. Much of the information is taken from earlier sections of the report and generally no attempt is made here to give full explanations. The working equations are all grouped together at the end of this section. A number of equations not given in the main text of this report have been included for completeness. The equations in this section are all designated by letters: (a), (b), (c), etc., and all references to equations in this section of the report are made in terms of these letters.

The instructions are given in the form most suitable for finding other quantities when \dot{m} , A , T_0 , and P are known. In many situations some of these quantities will be unknown and other quantities will be known. The user of the tables should in such instances have no great difficulty in inverting or otherwise revising the instructions given.

To Calculate ϕ . The first operation to be performed in using the tables is usually to find the force-variable ϕ ; as the first step

in finding ϕ , the sonic velocity c must be found. Equation (c) is used for this purpose, with a value of the constant C_1 taken from Table 2.

When c_* has been found, the force-variable ϕ is calculated from eq (e), using the appropriate value of C_2 taken from Table 3. In many applications it will be found that ϕ is equal to a constant times P and computations are expedited by first finding the constant and then multiplying by all the various values of P .

Using Table 4, Interpolation. When ϕ has been found, the corresponding values of λ , M , j , and all the other tabulated functions can be obtained from Table 4. Linear interpolation is to be used in all white areas of the table. In these areas the second differences of the tabulated functions are less than 4 in the last decimal place and the errors introduced by interpolation may be expected to be less than 1/2 in this place. For linear interpolation, eq (av) should be used. In the gray areas of Table 4, second differences are ≥ 4 and linear interpolation can introduce errors in the last decimal place or even preceding it. In the gray areas, 4-point interpolation is often satisfactory; for 4-point interpolation eq (aw) should be used. To verify the accuracy of 4-point interpolation, 5-point interpolation may be used. If the 4-point and 5-point interpolations agree, it may be assumed that no error is being introduced.

Alternatives to Interpolation. In many instances the procedures given in the table below are preferable to 4-point interpolation, being more convenient, more accurate, or both.

Desired Function	Recommended Method for Gray Areas
λ	eq (2)
j	eq (t)
T/T_o	eq (v)
c/c_*	eq (w)
P/P_o	eq (ac)
ρ/ρ_o	eq (ag) or (ah)
A/A_*	eq (am)

Of course a function can always be calculated to any desired accuracy from the equations used in the preparation of Table 4. Equations (t), (v), and (am) were in fact used in preparing Table 4. Unfortunately, many of the other primary equations are inconvenient to work with.

Other Operations. To find stagnation-pressure ratios (P_{02}/P_{01}), use either eq (ap) or eq (aq). The latter is more convenient when A_1/A_2 is constant. When $\ln (P_{02}/P_{01})$ is required, use eq (ar), getting the values of $\ln (A/A_*)$ from Table 4.

To solve Fanno-line or Rayleigh-line problems, use the general table subject to the restriction of eq (f) or eq (u), as the case may be.

To solve normal-shock problems, first find the desired properties of the flow before shock from Table 4. Then using eq (at), find the value of λ after a normal shock. Enter Table 4 at this value of λ and find the desired properties of the flow after the shock. Note that eqs (at) and (au) are equivalent and either one may be used.

Equations. The equations given below consist of the more useful ones from the main text plus a number of other useful equations that were not needed in the main text. The latter group of equations are designated only by letters. Where both a letter and a number appear, the number is the one used in the main text.

$$c = \sqrt{kRT} \quad (a) \quad (5)$$

$$c_* = \sqrt{\frac{2k}{k+1} R T_0} \quad (b) \quad (47)$$

$$c_* = C_1 \sqrt{T_0} \quad (c) \quad (48)$$

$$\phi = \frac{kPA}{c_* \dot{m}} \quad (d) \quad (1)$$

$$\phi = C_2 \frac{kPA}{c_* \dot{m}} \quad (e) \quad (49)$$

$$\phi = \text{constant} \times P \quad (f)$$

(Fanno-line flow)

$$\phi = \frac{k+1}{2} \frac{1}{\lambda} - \frac{k-1}{2} \lambda \quad (g) \quad (10)$$

$$\phi = \frac{k+1}{2} \frac{1}{\lambda} \frac{T}{T_0} \quad (h) \quad (8)$$

$$\phi = \frac{PA}{P_* A_*} \quad (i) \quad (13)$$

$$\lambda = \frac{u}{c_*} \quad (j)$$

$$\lambda = -\frac{\phi}{k-1} + \left[\left(\frac{\phi}{k-1} \right)^2 + \frac{k+1}{k-1} \right]^{\frac{1}{2}} \quad (k) \quad (19)$$

$$\lambda = \frac{\frac{k+1}{2}}{\phi + \frac{k-1}{2} \lambda} \quad (l) \quad (45)$$

$$\lambda = \frac{1}{k+1} + \left[\left(\frac{1}{k+1} \right)^2 - 1 \right]^{\frac{1}{2}} \quad (m) \quad (20)$$

$$\dot{m} = \rho u A \quad (n) \quad (2)$$

$$\dot{m} = \frac{PAu}{RT} \quad (o) \quad (3)$$

$$M = \frac{u}{c} \quad (p)$$

$$M = \frac{\lambda}{c/c_*} \quad (q) \quad (23)$$

$$j \equiv \frac{kJ}{c_* m} \quad (r) \quad (16)$$

$$j = \phi + k \lambda \quad (s) \quad (17)$$

$$j = \frac{k+1}{2} \left(\lambda + \frac{1}{\lambda} \right) \quad (t) \quad (18)$$

$$c_* j = \text{constant} \quad (u)$$

(Rayleigh-line flow)

$$\frac{T}{T_0} = 1 - \frac{k-1}{k+1} \lambda^2 \quad (v) \quad (9)$$

$$\frac{c}{c_*} = \sqrt{\phi \lambda} \quad (w) \quad (22)$$

$$\frac{c}{c_*} = \sqrt{\frac{T}{T_*}} \quad (x) \quad (21)$$

$$\frac{c}{c_*} = \sqrt{\frac{T_0}{T_*}} \sqrt{\frac{T}{T_0}} \quad (y) \quad (50)$$

$$= 1.0954 \sqrt{\frac{T}{T_0}} \quad (k=1.4) \quad (z)$$

$$\frac{P}{P_0} = \left(\frac{T}{T_0} \right)^{\frac{k}{k-1}} \quad (aa) \quad (25)$$

$$\frac{P}{P_0} = 1 - \frac{k}{k+1} \lambda^2 + \frac{1}{2} \frac{k}{k+1} \frac{1}{k+1} \lambda^4 - + \dots \quad (ab) \quad (46)$$

$$\approx 1 - 0.5833 \lambda^2 \quad (ac)$$

$$(k = 1.4, \text{ error} < 5 \times 10^{-5} \text{ for } 0 \leq \lambda \leq 0.142)$$

$$\ln \frac{P}{P_0} = - \left[\frac{k}{k+1} \lambda^2 + \frac{1}{2} \frac{k}{k+1} \frac{k-1}{k+1} \lambda^4 + \frac{1}{3} \frac{k}{k+1} \left(\frac{k-1}{k+1} \right)^2 \lambda^6 + \dots \right] \quad (\text{ad})$$

$$\frac{P}{P_0} = \left(\frac{T}{T_0} \right)^{\frac{1}{k-1}} \quad (\text{ae}) \quad (26)$$

$$= 1 - \frac{1}{k+1} \lambda^2 + \frac{1}{2} \frac{2-k}{(k+1)^2} \lambda^4 - + \dots \quad (\text{af})$$

$$\approx 1 - 0.4167 \lambda^2 \quad (\text{ag})$$

(k = 1.4, error < 5 x 10⁻⁵ for 0 ≤ λ ≤ 0.176)

$$\frac{P}{P_0} = \frac{P/P_0}{T/T_0} \quad (\text{ah})$$

$$\frac{P}{P_0} = \frac{P_*/P_0}{\lambda(A/A_*)} \quad (\text{ai}) \quad (29)$$

$$\frac{A}{A_*} = \frac{P_*}{P_0} \frac{1}{\lambda} \left(\frac{T}{T_0} \right)^{-\frac{1}{k-1}} \quad (\text{aj})$$

$$= \left(\frac{2}{k+1} \right)^{\frac{1}{k-1}} \left[\frac{1}{\lambda} + \frac{1}{k+1} \lambda + \frac{1}{2} \frac{k}{(k+1)^2} \lambda^3 + \dots \right] \quad (\text{ak})$$

$$\frac{A}{A_*} = \frac{P_*}{P_0} \frac{\phi}{P/P_0} \quad (\text{al}) \quad (28)$$

$$= 0.5283 \frac{\phi}{P/P_0} \quad (k = 1.4) \quad (\text{am})$$

$$\frac{4fL_*}{D} = \frac{k+1}{2k} \left[2 \ln \lambda + \frac{1}{\lambda^2} - 1 \right] \quad (\text{an}) \quad (43)$$

$$f = - \frac{\Delta(4fL_*/D)}{\Delta(4L/D)} \quad (\text{ao})$$

$$\frac{P_{o2}}{P_{o1}} = \frac{P_2 (P/P_o)_1}{P_1 (P/P_o)_2} \quad (\text{ap}) \quad (30)$$

$$\frac{P_{o2}}{P_{o1}} = \frac{A_1 (A/A_*)_2}{A_2 (A/A_*)_1} \quad (\text{aq}) \quad (33)$$

$$\ln \frac{P_{o2}}{P_{o1}} = \ln \frac{A_1}{A_2} + \ln \left(\frac{A}{A_*} \right)_2 - \ln \left(\frac{A}{A_*} \right)_1 \quad (\text{ar})$$

$$\frac{s_{o2} - s_{o1}}{R} = -\ln \frac{P_{o2}}{P_{o1}} \quad (T_o = \text{constant}) \quad (\text{as}) \quad (36)$$

$$\lambda_2 = \frac{1}{\lambda_1} \quad (\text{normal shock}) \quad (\text{at})$$

$$j_2 = j_1 \quad (\text{normal shock}) \quad (\text{au})$$

$$f = (1-p)f_o + pf_1 \quad (\text{av}) \quad (51)$$

(linear interpolation, p is fraction of interval)

$$f = A_1 f_{-1} + A_0 f_o + A_1 f_1 + A_2 f_2 \quad (\text{aw}) \quad (52)$$

(4-point interpolation, take A's from reference [8].)

6. Figures and Tables

The new functions are given in the present section of this report. They are shown graphically in Figs. 1, 2, and 3, and are given in tabular form in Table 4. Figures 1 and 2 show the various functions with the new variable ϕ as abscissa. Figure 3 gives the same information in a more conventional form, with M as abscissa. Tables 1, 2, and 3 are auxiliary tables containing various constants and conversion factors that are often needed when the main table (Table 4) is being used.

The methods used in calculating the functions are given in Section 3, and examples of the use of the tables are given in Section 7.

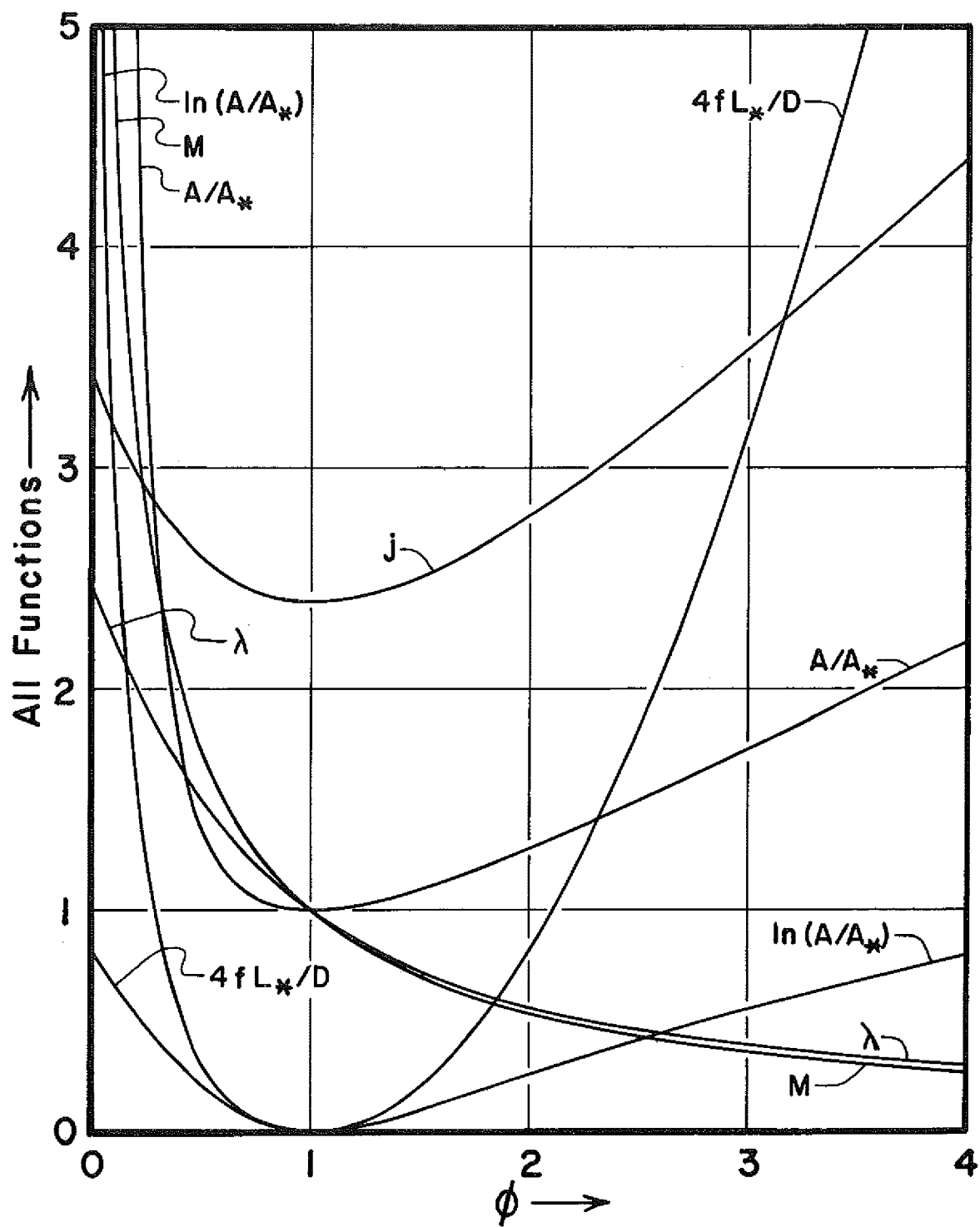


Fig. 1. Six of the tabulated functions (λ , M , j , A/A_* , and $4fL_*/D$) plotted versus ϕ .

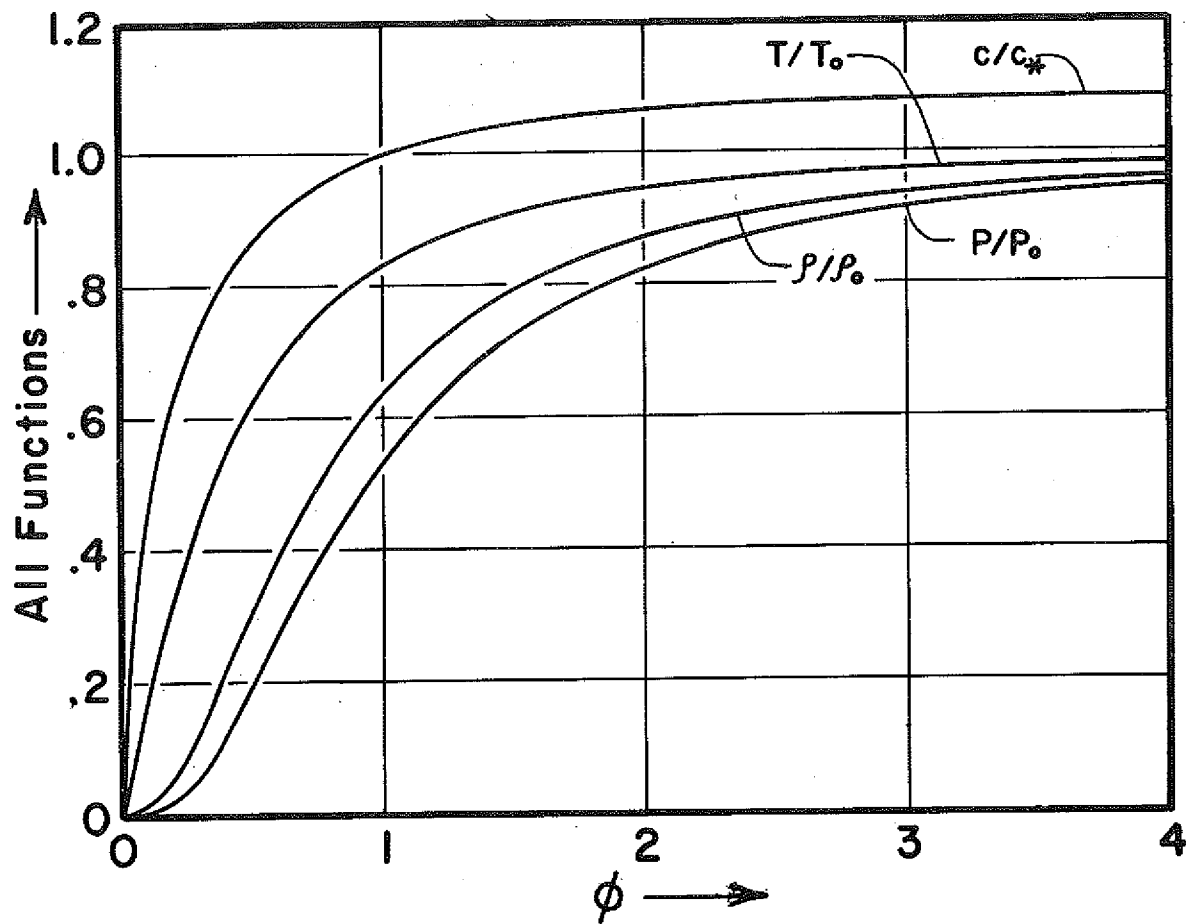


Fig. 2. Four of the tabulated functions (T/T_0 , c/c_* , P/P_0 , and ρ/ρ_0) plotted versus ϕ .

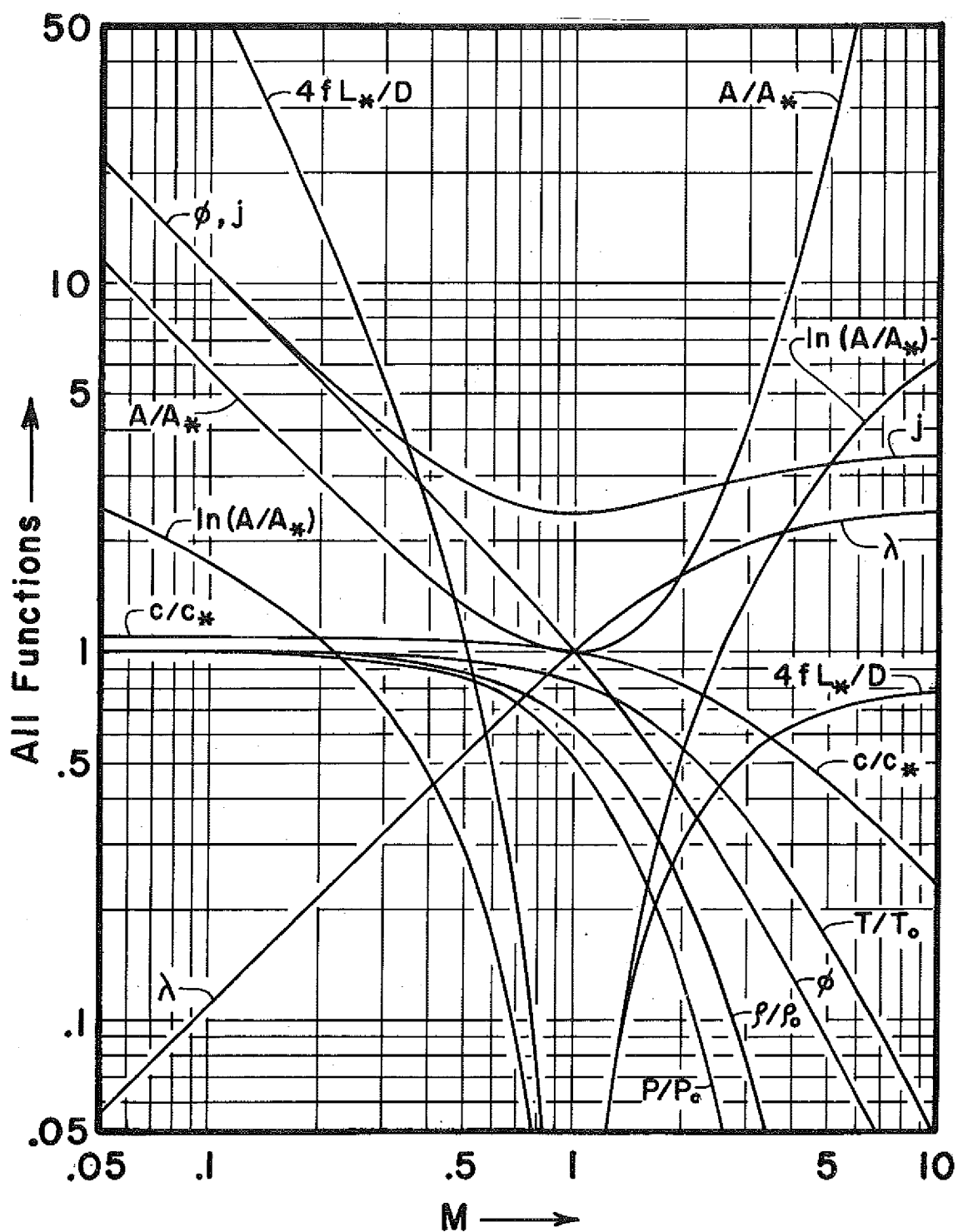


Fig. 3. Nine of the tabulated functions (λ , j , T/T_0 , c/c_* , P/P_* , ρ/ρ_0 , A/A_* , $\ln(A/A_*)$, and $4fL_*/D$) and ϕ plotted versus M .

Table 1. Constants and conversion factors *

$$\begin{aligned}
 R \text{ (all gases)} &= 8.3143 \text{ J/mol } ^\circ\text{K} \\
 \text{mol wt (air)} &= 28.970 \text{ g/mol} \\
 R \text{ (air)} &= 0.28700 \text{ J/g } ^\circ\text{K} \\
 &= 0.068594 \text{ thermochemical cal/g } ^\circ\text{K} \\
 &= 0.068594 \text{ thermochemical Btu/lbm } ^\circ\text{R} \\
 &= 53.342 \text{ ft lbf/lbm } ^\circ\text{R} \\
 &= 1716.2 \text{ ft}^2/\text{sec}^2 ^\circ\text{R} \\
 1 \text{ Btu/lbm} &= 25,020 \text{ ft}^2/\text{sec}^2 \\
 1 \text{ (Btu/lbm)}^{1/2} &= 158.18 \text{ ft/sec} \\
 &= 48.213 \text{ m/sec} \\
 1 \text{ atm} &= 760 \text{ mm Hg (Torr)} \\
 &= 14.696 \text{ psia} \\
 1 \text{ psia} &= 51.715 \text{ Torr} \\
 T(^{\circ}\text{R}) &= t(^{\circ}\text{F}) + 459.67 \\
 &= 1.8T(^{\circ}\text{K}) \\
 T(^{\circ}\text{K}) &= t(^{\circ}\text{C}) + 273.15 \\
 t(^{\circ}\text{F}) + 40 &= 1.8 [t(^{\circ}\text{C}) + 40]
 \end{aligned}$$

*Except for the molecular weight of air, which was taken from reference [1], all the values in this table were taken or calculated from reference [7], which uses the physical constants approved by the National Academy of Sciences and the National Bureau of Standards.

Table 2. Values of C_1 for use in the equation $c_* = C_1 \sqrt{T_o}$, when c_* and T_o are expressed in the indicated units.

T_o c_*	$^{\circ}R$ (= $^{\circ}F$ abs)	$^{\circ}K$
$\left(\frac{Btu}{lbm}\right)^{\frac{1}{2}}$	0.28289	0.37954
$\frac{ft}{sec}$	44,747	60.034
$\frac{m}{sec}$	13.639	18.298

Table 3. Values of C_2 required to make ϕ dimensionless in the equation

$$\phi = C_2 \frac{kPA}{c_* m}$$

Find the row headed by the dimensions of PA and the column headed by the dimensions of $c_* m$ and use the value of C_2 lying in the row and column thus found.

$c_* m$ PA	$\frac{cm}{sec} \frac{g}{sec}$	$\frac{ft}{sec} \frac{lbm}{sec}$	$\frac{ft}{sec} \frac{lbm}{hr}$	$\left(\frac{Btu}{lbm}\right)^{\frac{1}{2}} \frac{lbm}{sec}$	$\left(\frac{Btu}{lbm}\right)^{\frac{1}{2}} \frac{lbm}{hr}$
dyne	1	7.2330 $\times 10^{-5}$	0.26039	4.5727 $\times 10^{-7}$	1.6462 $\times 10^{-3}$
lbf	4.4482 $\times 10^5$	32.174 (= g)	1.1583 $\times 10^5$	0.20340	732.26
atm cm^2	1.0132 $\times 10^6$	73.289	2.6384 $\times 10^5$	0.46333	1668.0
atm in^2	6.5371 $\times 10^6$	472.83	1.7022 $\times 10^6$	2.9892	10,761
mm Hg cm^2	1333.2	0.096432	347.16	6.0965 $\times 10^{-4}$	2.1947
mm Hg in^2	8601.4	0.62214	2239.7	3.9332 $\times 10^{-3}$	14.159

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$).

ϕ	λ	M	J	T/T ₀	C/C*	P/P ₀	P/P ₀	A/A*	LN(A/A*)	4FL*/D	
GENERAL				ISOENERGETIC		ISENTHROPIC					FRICIONAL
1	2	3 INF	4	5	6	7	8	9 INF	10 INF	11	
0.00	2.4495	15.5712	3.4293	0.0000	0.0000	0.0000	0.0000	4505.5449	8.4131	0.8215	
0.01	2.4246	10.9545	3.4045	0.0202	0.1557	0.0000	0.0001	825.4375	6.7159	0.8070	
0.02	2.4000	8.8988	3.3800	0.0400	0.2191	0.0000	0.0003	310.4307	5.7380	0.7925	
0.03	2.3756	7.6673	3.3559	0.0594	0.2670	0.0001	0.0009	156.7190	5.0545	0.7780	
0.04	2.3515		3.3321	0.0784	0.3067	0.0001	0.0017			0.7637	
0.05	2.3277	6.8230	3.3087	0.0970	0.3412	0.0003	0.0029	92.9702	4.5323	0.7494	
0.06	2.3041	6.1969	3.2857	0.1152	0.3710	0.0005	0.0045	61.0773	4.1121	0.7352	
0.07	2.2807	5.7081	3.2630	0.1330	0.3996	0.0009	0.0065	43.0519	3.7624	0.7211	
0.08	2.2576	5.3123	3.2407	0.1505	0.4250	0.0013	0.0088	31.9508	3.4642	0.7070	
0.09	2.2348	4.9831	3.2187	0.1676	0.4485	0.0019	0.0115	24.6636	3.2053	0.6930	
0.10	2.2122	4.7034	3.1971	0.1844	0.4703	0.0027	0.0146	19.5383	2.9775	0.6791	
0.11	2.1899	4.4618	3.1758	0.2007	0.4908	0.0036	0.0181	16.0342	2.7747	0.6653	
0.12	2.1678	4.2503	3.1549	0.2169	0.5100	0.0047	0.0219	13.3654	2.5927	0.6516	
0.13	2.1460	4.0629	3.1343	0.2325	0.5282	0.0061	0.0261	11.3362	2.4280	0.6380	
0.14	2.1244	3.8954	3.1141	0.2478	0.5454	0.0076	0.0306	9.7583	2.2781	0.6245	
0.15	2.1030	3.7444	3.0942	0.2629	0.5617	0.0093	0.0354	8.5077	2.1410	0.6110	
0.16	2.0819	3.6072	3.0747	0.2776	0.5772	0.0113	0.0406	7.5060	2.0149	0.5977	
0.17	2.0611	3.4820	3.0555	0.2920	0.5919	0.0135	0.0461	6.6764	1.8986	0.5845	
0.18	2.0405	3.3669	3.0367	0.3061	0.6060	0.0159	0.0518	5.9945	1.7908	0.5713	
0.19	2.0201	3.2607	3.0182	0.3199	0.6195	0.0185	0.0579	5.4237	1.6908	0.5583	
0.20	2.0000	3.1623	3.0000	0.3333	0.6325	0.0214	0.0642	4.9410	1.5976	0.5454	
0.21	1.9801	3.0707	2.9822	0.3465	0.6448	0.0245	0.0707	4.5293	1.5106	0.5326	
0.22	1.9605	2.9852	2.9647	0.3594	0.6567	0.0278	0.0774	4.1752	1.4292	0.5199	
0.23	1.9411	2.9051	2.9475	0.3720	0.6682	0.0314	0.0844	3.8684	1.3528	0.5073	
0.24	1.9219	2.8298	2.9307	0.3844	0.6792	0.0352	0.0916	3.6009	1.2812	0.4949	
0.25	1.9030	2.7590	2.9142	0.3965	0.6897	0.0392	0.0990	3.3662	1.2138	0.4825	
0.26	1.8843	2.6921	2.8980	0.4083	0.6999	0.0435	0.1065	3.1591	1.1505	0.4703	
0.27	1.8658	2.6288	2.8821	0.4198	0.7098	0.0479	0.1142	2.9756	1.0904	0.4583	
0.28	1.8475	2.5687	2.8666	0.4311	0.7192	0.0526	0.1220	2.8120	1.0339	0.4463	
0.29	1.8295	2.5117	2.8513	0.4421	0.7284	0.0575	0.1300	2.6657	0.9805	0.4345	

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

ϕ	λ	M	J	T/T ₀	C/C*	P/P ₀	P/P_0	A/A*	LN(A/A*)	4FL*/D	
GENERAL				ISOENERGETIC		ISENTROPIC					FRICITIONAL
0.30	1.8117	2.4575	2.8364	0.4529	0.7372	0.0625	0.1381	2.5343	0.9299	0.4228	
0.31	1.7942	2.4058	2.8218	0.4635	0.7458	0.0678	0.1463	2.4159	0.8821	0.4112	
0.32	1.7768	2.3564	2.8075	0.4738	0.7540	0.0732	0.1545	2.3067	0.8367	0.3998	
0.33	1.7597	2.3092	2.7936	0.4839	0.7620	0.0788	0.1629	2.2115	0.7937	0.3885	
0.34	1.7428	2.2640	2.7799	0.4938	0.7698	0.0846	0.1713	2.1230	0.7528	0.3773	
0.35	1.7261	2.2207	2.7665	0.5034	0.7773	0.0905	0.1798	2.0423	0.7141	0.3663	
0.36	1.7096	2.1792	2.7534	0.5129	0.7845	0.0966	0.1884	1.9684	0.6772	0.3554	
0.37	1.6933	2.1393	2.7407	0.5221	0.7915	0.1028	0.1970	1.9007	0.6422	0.3447	
0.38	1.6773	2.1009	2.7282	0.5311	0.7983	0.1092	0.2056	1.8384	0.6089	0.3341	
0.39	1.6614	2.0640	2.7160	0.5400	0.8050	0.1157	0.2142	1.7810	0.5772	0.3237	
0.40	1.6458	2.0284	2.7041	0.5486	0.8114	0.1223	0.2229	1.7281	0.5479	0.3134	
0.41	1.6303	1.9941	2.6924	0.5570	0.8176	0.1290	0.2316	1.6792	0.5183	0.3032	
0.42	1.6151	1.9610	2.6811	0.5653	0.8236	0.1358	0.2402	1.6339	0.4919	0.2932	
0.43	1.6000	1.9290	2.6700	0.5733	0.8295	0.1427	0.2489	1.5919	0.4649	0.2834	
0.44	1.5851	1.8981	2.6592	0.5812	0.8351	0.1497	0.2575	1.5528	0.4401	0.2737	
0.45	1.5705	1.8681	2.6487	0.5889	0.8407	0.1568	0.2662	1.5165	0.4164	0.2642	
0.46	1.5560	1.8392	2.6384	0.5965	0.8460	0.1639	0.2748	1.4827	0.3939	0.2548	
0.47	1.5417	1.8112	2.6284	0.6038	0.8512	0.1711	0.2833	1.4512	0.3724	0.2456	
0.48	1.5276	1.7840	2.6187	0.6111	0.8563	0.1784	0.2919	1.4218	0.3519	0.2365	
0.49	1.5137	1.7576	2.6092	0.6181	0.8612	0.1857	0.3004	1.3943	0.3324	0.2276	
0.50	1.5000	1.7321	2.6000	0.6250	0.8660	0.1930	0.3088	1.3685	0.3137	0.2189	
0.51	1.4865	1.7072	2.5910	0.6317	0.8707	0.2004	0.3172	1.3444	0.2960	0.2103	
0.52	1.4731	1.6831	2.5823	0.6383	0.8752	0.2078	0.3256	1.3219	0.2791	0.2019	
0.53	1.4599	1.6597	2.5738	0.6448	0.8796	0.2153	0.3338	1.3007	0.2629	0.1937	
0.54	1.4469	1.6369	2.5656	0.6511	0.8839	0.2227	0.3421	1.2809	0.2475	0.1856	
0.55	1.4340	1.6147	2.5576	0.6573	0.8881	0.2302	0.3502	1.2622	0.2329	0.1776	
0.56	1.4213	1.5931	2.5499	0.6633	0.8922	0.2377	0.3583	1.2447	0.2189	0.1699	
0.57	1.4088	1.5721	2.5424	0.6692	0.8961	0.2452	0.3663	1.2283	0.2056	0.1623	
0.58	1.3965	1.5517	2.5351	0.6750	0.9000	0.2526	0.3743	1.2128	0.1930	0.1549	
0.59	1.3843	1.5318	2.5280	0.6806	0.9037	0.2601	0.3822	1.1983	0.1809	0.1476	

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

ϕ	λ	M	J	T/T ₀	C/C*	P/P ₀	P/P*	A/A*	LN(A/A*)	4FL*/D
GENERAL				ISOENERGETIC			ISENTHROPIC			FRICTIONAL
0.60	1.3723	1.5123	2.5212	0.6861	0.9074	0.2676	0.3900	1.1846	0.1694	0.1405
0.61	1.3604	1.4984	2.5146	0.6915	0.9110	0.2750	0.3977	1.1717	0.1585	0.1336
0.62	1.3487	1.4749	2.5082	0.6968	0.9144	0.2825	0.4053	1.1596	0.1481	0.1269
0.63	1.3372	1.4569	2.5020	0.7020	0.9178	0.2899	0.4129	1.1482	0.1382	0.1203
0.64	1.3257	1.4393	2.4960	0.7071	0.9211	0.2972	0.4204	1.1375	0.1288	0.1139
0.65	1.3145	1.4221	2.4903	0.7120	0.9243	0.3046	0.4278	1.1274	0.1199	0.1077
0.66	1.3034	1.4053	2.4847	0.7169	0.9275	0.3119	0.4351	1.1178	0.1114	0.1016
0.67	1.2924	1.3889	2.4794	0.7216	0.9306	0.3192	0.4423	1.1089	0.1034	0.0958
0.68	1.2816	1.3729	2.4743	0.7262	0.9335	0.3264	0.4495	1.1005	0.0957	0.0900
0.69	1.2709	1.3572	2.4693	0.7308	0.9365	0.3336	0.4565	1.0926	0.0885	0.0845
0.70	1.2604	1.3419	2.4646	0.7352	0.9393	0.3408	0.4635	1.0851	0.0817	0.0791
0.71	1.2500	1.3269	2.4600	0.7396	0.9421	0.3479	0.4704	1.0781	0.0752	0.0740
0.72	1.2397	1.3122	2.4556	0.7438	0.9448	0.3550	0.4772	1.0716	0.0691	0.0689
0.73	1.2296	1.2978	2.4515	0.7480	0.9474	0.3620	0.4839	1.0654	0.0633	0.0641
0.74	1.2196	1.2838	2.4475	0.7521	0.9500	0.3689	0.4905	1.0596	0.0579	0.0594
0.75	1.2097	1.2700	2.4436	0.7561	0.9525	0.3758	0.4971	1.0542	0.0528	0.0550
0.76	1.2000	1.2566	2.4400	0.7600	0.9550	0.3827	0.5035	1.0491	0.0480	0.0506
0.77	1.1904	1.2434	2.4365	0.7638	0.9574	0.3895	0.5099	1.0444	0.0434	0.0465
0.78	1.1809	1.2304	2.4333	0.7676	0.9597	0.3962	0.5162	1.0400	0.0392	0.0425
0.79	1.1715	1.2178	2.4301	0.7713	0.9620	0.4029	0.5224	1.0359	0.0352	0.0388
0.80	1.1623	1.2053	2.4272	0.7749	0.9643	0.4095	0.5285	1.0320	0.0315	0.0352
0.81	1.1531	1.1932	2.4244	0.7784	0.9665	0.4161	0.5345	1.0285	0.0281	0.0317
0.82	1.1441	1.1812	2.4218	0.7818	0.9686	0.4226	0.5405	1.0252	0.0249	0.0285
0.83	1.1352	1.1695	2.4193	0.7852	0.9707	0.4290	0.5463	1.0221	0.0219	0.0254
0.84	1.1265	1.1580	2.4170	0.7885	0.9727	0.4354	0.5521	1.0193	0.0191	0.0225
0.85	1.1178	1.1468	2.4149	0.7918	0.9747	0.4417	0.5578	1.0167	0.0166	0.0198
0.86	1.1092	1.1357	2.4129	0.7949	0.9767	0.4479	0.5634	1.0144	0.0143	0.0172
0.87	1.1008	1.1248	2.4111	0.7981	0.9786	0.4541	0.5690	1.0122	0.0121	0.0148
0.88	1.0924	1.1142	2.4094	0.8011	0.9805	0.4602	0.5744	1.0103	0.0102	0.0126
0.89	1.0842	1.1037	2.4078	0.8041	0.9823	0.4662	0.5798	1.0085	0.0085	0.0106

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

ϕ	λ	M	J	T/T ₀	C/C*	P/P ₀	P/P ₀	A/A*	LN(A/A*)	4FL*/D	
GENERAL				ISOENERGETIC		ISENTROPIC					FRICIONAL
0.90	1.0760	1.0934	2.4064	0.8070	0.9841	0.4722	0.5851	1.0069	0.0069	0.0088	
0.91	1.0680	1.0833	2.4052	0.8099	0.9858	0.4781	0.5903	1.0055	0.0055	0.0071	
0.92	1.0601	1.0734	2.4041	0.8127	0.9875	0.4839	0.5954	1.0043	0.0043	0.0056	
0.93	1.0522	1.0637	2.4031	0.8155	0.9892	0.4897	0.6005	1.0033	0.0033	0.0043	
0.94	1.0445	1.0541	2.4023	0.8182	0.9909	0.4954	0.6055	1.0024	0.0024	0.0032	
0.95	1.0368	1.0447	2.4016	0.8208	0.9925	0.5011	0.6104	1.0016	0.0016	0.0022	
0.96	1.0293	1.0355	2.4010	0.8234	0.9940	0.5066	0.6153	1.0010	0.0010	0.0014	
0.97	1.0218	1.0264	2.4006	0.8260	0.9956	0.5121	0.6200	1.0006	0.0006	0.0008	
0.98	1.0145	1.0174	2.4002	0.8285	0.9971	0.5176	0.6247	1.0002	0.0002	0.0004	
0.99	1.0072	1.0086	2.4001	0.8309	0.9986	0.5230	0.6294	1.0001	0.0001	0.0001	
1.00	1.0000	1.0000	2.4000	0.8333	1.0000	0.5283	0.6339	1.0000	0.0000	0.0000	
1.01	0.9929	0.9915	2.4001	0.8357	1.0014	0.5335	0.6384	1.0001	0.0001	0.0001	
1.02	0.9859	0.9831	2.4002	0.8380	1.0028	0.5387	0.6429	1.0002	0.0002	0.0003	
1.03	0.9790	0.9749	2.4005	0.8403	1.0042	0.5438	0.6472	1.0005	0.0005	0.0008	
1.04	0.9721	0.9668	2.4010	0.8425	1.0055	0.5489	0.6515	1.0009	0.0009	0.0014	
1.05	0.9654	0.9588	2.4015	0.8447	1.0068	0.5539	0.6557	1.0014	0.0014	0.0022	
1.06	0.9587	0.9510	2.4021	0.8468	1.0081	0.5588	0.6599	1.0021	0.0021	0.0031	
1.07	0.9521	0.9433	2.4029	0.8489	1.0093	0.5637	0.6640	1.0028	0.0028	0.0043	
1.08	0.9455	0.9357	2.4038	0.8510	1.0105	0.5685	0.6681	1.0036	0.0036	0.0056	
1.09	0.9391	0.9282	2.4047	0.8530	1.0117	0.5733	0.6720	1.0045	0.0045	0.0071	
1.10	0.9327	0.9208	2.4058	0.8550	1.0129	0.5779	0.6760	1.0055	0.0055	0.0087	
1.11	0.9264	0.9136	2.4070	0.8570	1.0141	0.5826	0.6798	1.0066	0.0065	0.0105	
1.12	0.9202	0.9064	2.4083	0.8589	1.0152	0.5871	0.6836	1.0077	0.0077	0.0125	
1.13	0.9141	0.8994	2.4097	0.8607	1.0163	0.5916	0.6874	1.0090	0.0089	0.0147	
1.14	0.9080	0.8925	2.4112	0.8626	1.0174	0.5961	0.6911	1.0103	0.0102	0.0170	
1.15	0.9020	0.8856	2.4128	0.8644	1.0185	0.6005	0.6947	1.0117	0.0116	0.0196	
1.16	0.8961	0.8789	2.4145	0.8662	1.0195	0.6048	0.6983	1.0132	0.0131	0.0222	
1.17	0.8902	0.8723	2.4163	0.8679	1.0205	0.6091	0.7018	1.0147	0.0146	0.0251	
1.18	0.8844	0.8657	2.4181	0.8696	1.0216	0.6133	0.7053	1.0164	0.0162	0.0281	
1.19	0.8787	0.8593	2.4201	0.8713	1.0225	0.6175	0.7087	1.0181	0.0179	0.0313	

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

ϕ	λ	M	J	T/T ₀	C/C*	P/P ₀	P/P ₀	A/A*	LN(A/A*)	4FL*/D	
GENERAL				ISOENERGETIC		ISENTROPIC					FRICTIONAL
1.20	0.8730	0.8529	2.4222	0.8730	1.0235	0.6216	0.7121	1.0198	0.0196	0.0347	
1.22	0.8618	0.8405	2.4266	0.8762	1.0254	0.6297	0.7186	1.0235	0.0233	0.0420	
1.24	0.8509	0.8284	2.4313	0.8793	1.0272	0.6375	0.7250	1.0275	0.0271	0.0499	
1.26	0.8403	0.8166	2.4364	0.8823	1.0290	0.6452	0.7312	1.0317	0.0312	0.0585	
1.28	0.8299	0.8052	2.4418	0.8852	1.0307	0.6526	0.7373	1.0361	0.0355	0.0678	
1.30	0.8197	0.7941	2.4476	0.8880	1.0323	0.6599	0.7431	1.0407	0.0399	0.0777	
1.32	0.8097	0.7832	2.4536	0.8907	1.0339	0.6669	0.7488	1.0456	0.0445	0.0883	
1.34	0.8000	0.7727	2.4600	0.8933	1.0354	0.6738	0.7543	1.0506	0.0493	0.0996	
1.36	0.7905	0.7624	2.4667	0.8959	1.0368	0.6805	0.7596	1.0558	0.0543	0.1116	
1.38	0.7811	0.7524	2.4736	0.8983	1.0383	0.6870	0.7648	1.0611	0.0593	0.1242	
1.40	0.7720	0.7426	2.4808	0.9007	1.0396	0.6934	0.7699	1.0666	0.0645	0.1374	
1.42	0.7631	0.7331	2.4883	0.9030	1.0409	0.6996	0.7748	1.0723	0.0698	0.1514	
1.44	0.7543	0.7238	2.4960	0.9052	1.0422	0.7056	0.7795	1.0781	0.0752	0.1660	
1.46	0.7457	0.7147	2.5040	0.9073	1.0434	0.7115	0.7841	1.0841	0.0807	0.1812	
1.48	0.7373	0.7058	2.5123	0.9094	1.0446	0.7172	0.7886	1.0902	0.0864	0.1971	
1.50	0.7291	0.6972	2.5208	0.9114	1.0458	0.7227	0.7930	1.0964	0.0921	0.2135	
1.52	0.7211	0.6888	2.5295	0.9133	1.0469	0.7282	0.7972	1.1028	0.0978	0.2306	
1.54	0.7132	0.6805	2.5384	0.9152	1.0480	0.7334	0.8014	1.1092	0.1037	0.2486	
1.56	0.7054	0.6725	2.5475	0.9171	1.0490	0.7386	0.8054	1.1158	0.1096	0.2671	
1.58	0.6978	0.6646	2.5570	0.9188	1.0500	0.7436	0.8093	1.1225	0.1156	0.2862	
1.60	0.6904	0.6569	2.5666	0.9206	1.0510	0.7485	0.8131	1.1293	0.1216	0.3060	
1.62	0.6831	0.6494	2.5764	0.9222	1.0520	0.7532	0.8168	1.1362	0.1277	0.3263	
1.64	0.6760	0.6420	2.5864	0.9238	1.0529	0.7579	0.8203	1.1432	0.1338	0.3473	
1.66	0.6690	0.6348	2.5966	0.9254	1.0538	0.7624	0.8238	1.1503	0.1400	0.3690	
1.68	0.6621	0.6278	2.6069	0.9269	1.0547	0.7668	0.8272	1.1574	0.1462	0.3913	
1.70	0.6554	0.6209	2.6175	0.9284	1.0555	0.7711	0.8305	1.1647	0.1525	0.4142	
1.72	0.6487	0.6141	2.6282	0.9299	1.0563	0.7753	0.8338	1.1720	0.1587	0.4377	
1.74	0.6422	0.6075	2.6391	0.9313	1.0571	0.7794	0.8369	1.1794	0.1650	0.4618	
1.76	0.6359	0.6011	2.6502	0.9326	1.0579	0.7833	0.8400	1.1869	0.1714	0.4866	
1.78	0.6296	0.5947	2.6615	0.9339	1.0586	0.7872	0.8429	1.1945	0.1777	0.5120	

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

ϕ	λ	M	J	T/T ₀	C/C*	P/P ₀	P/P ₀	A/A*	LN(A/A*)	4FL*/D	
GENERAL				ISOENERGETIC		ISENTHROPIC					FRICTIONAL
1.80	0.6235	0.5885	2.6729	0.9352	1.0594	0.7910	0.8458	1.2021	0.1841	0.5380	
1.82	0.6174	0.5825	2.6844	0.9365	1.0601	0.7947	0.8486	1.2098	0.1905	0.5446	
1.84	0.6115	0.5765	2.6961	0.9377	1.0608	0.7983	0.8514	1.2176	0.1969	0.5518	
1.86	0.6057	0.5707	2.7080	0.9389	1.0614	0.8018	0.8541	1.2254	0.2033	0.5597	
1.88	0.6000	0.5649	2.7200	0.9400	1.0621	0.8053	0.8567	1.2333	0.2097	0.5661	
1.90	0.5944	0.5593	2.7321	0.9411	1.0627	0.8086	0.8592	1.2413	0.2161	0.5722	
1.92	0.5889	0.5538	2.7444	0.9422	1.0633	0.8119	0.8617	1.2493	0.2226	0.5788	
1.94	0.5835	0.5484	2.7568	0.9433	1.0639	0.8151	0.8641	1.2573	0.2290	0.5851	
1.96	0.5781	0.5431	2.7694	0.9443	1.0645	0.8182	0.8665	1.2655	0.2354	0.5917	
1.98	0.5729	0.5379	2.7821	0.9453	1.0651	0.8213	0.8688	1.2736	0.2419	0.5984	
2.00	0.5678	0.5328	2.7949	0.9463	1.0656	0.8243	0.8711	1.2818	0.2483	0.6051	
2.02	0.5627	0.5278	2.8078	0.9472	1.0661	0.8272	0.8732	1.2901	0.2547	0.6118	
2.04	0.5577	0.5229	2.8208	0.9482	1.0667	0.8300	0.8754	1.2984	0.2612	0.6184	
2.06	0.5529	0.5180	2.8340	0.9491	1.0672	0.8328	0.8775	1.3068	0.2676	0.6251	
2.08	0.5480	0.5133	2.8473	0.9499	1.0677	0.8355	0.8795	1.3152	0.2740	0.6317	
2.10	0.5433	0.5086	2.8606	0.9508	1.0682	0.8381	0.8815	1.3236	0.2804	0.6384	
2.12	0.5387	0.5041	2.8741	0.9516	1.0686	0.8407	0.8835	1.3321	0.2868	0.6451	
2.14	0.5341	0.4996	2.8877	0.9525	1.0691	0.8433	0.8854	1.3407	0.2932	0.6518	
2.16	0.5296	0.4952	2.9014	0.9533	1.0695	0.8457	0.8872	1.3492	0.2995	0.6584	
2.18	0.5252	0.4908	2.9152	0.9540	1.0700	0.8482	0.8890	1.3578	0.3059	0.6651	
2.20	0.5208	0.4865	2.9291	0.9548	1.0704	0.8505	0.8908	1.3665	0.3122	0.6718	
2.22	0.5165	0.4823	2.9431	0.9555	1.0708	0.8528	0.8925	1.3752	0.3186	0.6784	
2.24	0.5123	0.4782	2.9572	0.9563	1.0712	0.8551	0.8942	1.3839	0.3249	0.6851	
2.26	0.5081	0.4742	2.9714	0.9570	1.0716	0.8573	0.8959	1.3926	0.3312	0.6918	
2.28	0.5040	0.4702	2.9856	0.9577	1.0720	0.8595	0.8975	1.4014	0.3375	0.6984	
2.30	0.5000	0.4663	3.0000	0.9583	1.0724	0.8616	0.8991	1.4102	0.3437	0.7051	
2.32	0.4960	0.4624	3.0144	0.9590	1.0727	0.8637	0.9006	1.4191	0.3500	0.7118	
2.34	0.4921	0.4586	3.0290	0.9596	1.0731	0.8657	0.9021	1.4279	0.3562	0.7184	
2.36	0.4883	0.4549	3.0436	0.9603	1.0735	0.8677	0.9036	1.4368	0.3624	0.7251	
2.38	0.4845	0.4512	3.0583	0.9609	1.0738	0.8696	0.9051	1.4458	0.3686	0.7318	

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

ϕ	λ	M	J	ISOENERGETIC			ISENTHROPIC			4FL*/D
				T/T ₀	C/C*	P/P ₀	P/P ₀	A/A*	LN(A/A*)	
GENERAL										
2.40	0.4807	0.4476	3.0730	0.9615	1.0741	0.8716	0.9065	1.4547	0.3748	1.5961
2.42	0.4771	0.4440	3.0879	0.9621	1.0745	0.8734	0.9079	1.4637	0.3810	1.6403
2.44	0.4734	0.4405	3.1028	0.9626	1.0748	0.8752	0.9092	1.4727	0.3871	1.6852
2.46	0.4699	0.4370	3.1178	0.9632	1.0751	0.8770	0.9105	1.4818	0.3932	1.7306
2.48	0.4663	0.4336	3.1329	0.9638	1.0754	0.8788	0.9118	1.4908	0.3993	1.7766
2.50	0.4629	0.4303	3.1480	0.9643	1.0757	0.8805	0.9131	1.4999	0.4054	1.8231
2.55	0.4544	0.4221	3.1862	0.9656	1.0764	0.8846	0.9162	1.5228	0.4205	1.9420
2.60	0.4462	0.4143	3.2247	0.9668	1.0771	0.8886	0.9191	1.5457	0.4355	2.0643
2.65	0.4383	0.4067	3.2637	0.9680	1.0778	0.8923	0.9219	1.5688	0.4503	2.1901
2.70	0.4307	0.3994	3.3030	0.9691	1.0784	0.8959	0.9245	1.5921	0.4650	2.3194
2.75	0.4233	0.3923	3.3427	0.9701	1.0790	0.8993	0.9270	1.6154	0.4796	2.4522
2.80	0.4162	0.3855	3.3827	0.9711	1.0795	0.9025	0.9294	1.6389	0.4940	2.5884
2.85	0.4093	0.3790	3.4230	0.9721	1.0800	0.9056	0.9317	1.6625	0.5083	2.7290
2.90	0.4026	0.3726	3.4637	0.9730	1.0805	0.9086	0.9338	1.6861	0.5224	2.8730
2.95	0.3961	0.3664	3.5046	0.9738	1.0810	0.9114	0.9359	1.7099	0.5364	3.0175
3.00	0.3899	0.3605	3.5458	0.9747	1.0815	0.9141	0.9379	1.7338	0.5503	3.1673
3.05	0.3838	0.3547	3.5873	0.9755	1.0819	0.9167	0.9398	1.7577	0.5640	3.3205
3.10	0.3779	0.3491	3.6290	0.9762	1.0823	0.9192	0.9416	1.7817	0.5776	3.4771
3.15	0.3722	0.3437	3.6710	0.9769	1.0827	0.9215	0.9433	1.8058	0.5910	3.6371
3.20	0.3666	0.3385	3.7132	0.9776	1.0831	0.9238	0.9449	1.8300	0.6043	3.8004
3.25	0.3612	0.3334	3.7557	0.9783	1.0835	0.9259	0.9465	1.8542	0.6175	3.9670
3.30	0.3560	0.3284	3.7983	0.9789	1.0838	0.9280	0.9480	1.8785	0.6305	4.1369
3.35	0.3509	0.3236	3.8412	0.9795	1.0841	0.9300	0.9495	1.9029	0.6434	4.3112
3.40	0.3459	0.3190	3.8843	0.9801	1.0845	0.9319	0.9509	1.9274	0.6561	4.4868
3.45	0.3411	0.3144	3.9275	0.9806	1.0848	0.9338	0.9522	1.9518	0.6688	4.6667
3.50	0.3364	0.3100	3.9709	0.9811	1.0851	0.9355	0.9535	1.9764	0.6813	4.8499
3.55	0.3318	0.3057	4.0146	0.9816	1.0853	0.9372	0.9548	2.0010	0.6936	5.0363
3.60	0.3274	0.3016	4.0583	0.9821	1.0856	0.9389	0.9559	2.0256	0.7059	5.2261
3.65	0.3230	0.2975	4.1023	0.9826	1.0859	0.9404	0.9571	2.0504	0.7180	5.4191
3.70	0.3188	0.2935	4.1464	0.9831	1.0861	0.9419	0.9582	2.0751	0.7300	5.6154

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

ϕ	λ	M	J	T/T ₀	C/C*	P/P ₀	P/P ₀	A/A*	LN(A/A*)	4FL*/D	
GENERAL				ISOENERGETIC		ISENTHROPIC					FRICIONAL
3.75	0.3147	0.2897	4.1906	0.9835	1.0864	0.9434	0.9592	2.0999	0.7419	5.8149	
3.80	0.3107	0.2859	4.2350	0.9839	1.0866	0.9448	0.9603	2.1247	0.7536	6.0177	
3.85	0.3068	0.2823	4.2795	0.9843	1.0868	0.9462	0.9612	2.1496	0.7653	6.2237	
3.90	0.3030	0.2787	4.3242	0.9847	1.0870	0.9475	0.9622	2.1745	0.7768	6.4330	
3.95	0.2993	0.2753	4.3690	0.9851	1.0872	0.9487	0.9631	2.1995	0.7882	6.6455	
4.0	0.2956	0.2719	4.4139	0.9854	1.0874	0.9499	0.9640	2.2245	0.7995	6.8612	
4.1	0.2886	0.2653	4.5041	0.9861	1.0878	0.9522	0.9657	2.2746	0.8218	7.3023	
4.2	0.2819	0.2591	4.5947	0.9868	1.0882	0.9544	0.9672	2.3248	0.8436	7.7562	
4.3	0.2755	0.2531	4.6853	0.9873	1.0885	0.9564	0.9687	2.3751	0.8651	8.2230	
4.4	0.2694	0.2475	4.7772	0.9879	1.0888	0.9583	0.9700	2.4256	0.8861	8.7024	
4.5	0.2636	0.2420	4.8690	0.9884	1.0891	0.9601	0.9713	2.4762	0.9067	9.1947	
4.6	0.2580	0.2368	4.9612	0.9889	1.0894	0.9617	0.9725	2.5268	0.9270	9.6996	
4.7	0.2526	0.2318	5.0536	0.9894	1.0896	0.9633	0.9736	2.5776	0.9469	10.2171	
4.8	0.2474	0.2271	5.1464	0.9898	1.0898	0.9647	0.9747	2.6284	0.9664	10.7473	
4.9	0.2425	0.2225	5.2395	0.9902	1.0901	0.9661	0.9757	2.6794	0.9856	11.2901	
5.0	0.2377	0.2181	5.3328	0.9906	1.0903	0.9674	0.9766	2.7304	1.0044	11.8435	
5.1	0.2332	0.2138	5.4264	0.9909	1.0905	0.9686	0.9775	2.7814	1.0230	12.4134	
5.2	0.2288	0.2097	5.5203	0.9913	1.0907	0.9698	0.9783	2.8326	1.0412	12.9938	
5.3	0.2245	0.2058	5.6143	0.9916	1.0908	0.9709	0.9791	2.8838	1.0591	13.5868	
5.4	0.2204	0.2020	5.7086	0.9919	1.0910	0.9719	0.9799	2.9351	1.0767	14.1922	
5.5	0.2165	0.1984	5.8031	0.9922	1.0912	0.9729	0.9806	2.9864	1.0941	14.8101	
5.6	0.2127	0.1949	5.8977	0.9925	1.0913	0.9739	0.9813	3.0378	1.1111	15.4404	
5.7	0.2090	0.1915	5.9926	0.9927	1.0915	0.9748	0.9819	3.0892	1.1279	16.0832	
5.8	0.2054	0.1882	6.0876	0.9930	1.0916	0.9756	0.9825	3.1407	1.1444	16.7384	
5.9	0.2020	0.1850	6.1828	0.9932	1.0917	0.9764	0.9831	3.1922	1.1607	17.4059	
6.0	0.1987	0.1820	6.2782	0.9934	1.0918	0.9772	0.9836	3.2438	1.1767	18.0859	
6.1	0.1955	0.1790	6.3737	0.9936	1.0920	0.9779	0.9842	3.2954	1.1925	18.7781	
6.2	0.1924	0.1761	6.4693	0.9938	1.0921	0.9786	0.9847	3.3470	1.2081	19.4828	
6.3	0.1893	0.1734	6.5651	0.9940	1.0922	0.9792	0.9851	3.3987	1.2234	20.1998	
6.4	0.1864	0.1707	6.6610	0.9942	1.0923	0.9799	0.9856	3.4504	1.2385	20.9291	

Table 4. Functions for the 1-dimensional flow of air ($k=1.4$), continued.

φ	λ	M	J	GENERAL			ISOENERGETIC			ISENTHROPIC			LN(A/A*)	4FL*/D
				T/T0	C/C*	P/P0	P/P0	A/A*	FRICIONAL					
6.5	0.1836	0.1681	6.7570	0.9944	1.0924	0.9805	0.9860	3.5022	1.2534	21.6707				
6.6	0.1808	0.1655	6.8532	0.9946	1.0925	0.9811	0.9864	3.5540	1.2681	22.4256				
6.7	0.1782	0.1631	6.9494	0.9947	1.0925	0.9816	0.9868	3.6058	1.2825	23.1908				
6.8	0.1756	0.1607	7.0458	0.9949	1.0926	0.9821	0.9872	3.6576	1.2968	23.9692				
6.9	0.1730	0.1584	7.1423	0.9950	1.0927	0.9826	0.9876	3.7095	1.3109	24.7599				
7.0	0.1706	0.1561	7.2388	0.9951	1.0928	0.9831	0.9879	3.7614	1.3248	25.5629				
7.1	0.1682	0.1539	7.3355	0.9953	1.0929	0.9836	0.9883	3.8134	1.3385	26.3781				
7.2	0.1659	0.1518	7.4323	0.9954	1.0929	0.9840	0.9886	3.8653	1.3520	27.2036				
7.3	0.1636	0.1497	7.5291	0.9955	1.0930	0.9845	0.9889	3.9173	1.3654	28.0453				
7.4	0.1615	0.1477	7.6260	0.9957	1.0931	0.9849	0.9892	3.9693	1.3786	28.8972				
7.5	0.1593	0.1458	7.7231	0.9958	1.0931	0.9853	0.9895	4.0213	1.3916	29.7613				
7.6	0.1572	0.1438	7.8201	0.9959	1.0932	0.9857	0.9897	4.0734	1.4045	30.6375				
7.7	0.1552	0.1420	7.9173	0.9960	1.0932	0.9860	0.9900	4.1254	1.4172	31.5261				
7.8	0.1532	0.1402	8.0145	0.9961	1.0933	0.9864	0.9903	4.1775	1.4297	32.4268				
7.9	0.1513	0.1384	8.1118	0.9962	1.0934	0.9867	0.9905	4.2296	1.4421	33.3398				
8.0	0.1494	0.1367	8.2092	0.9963	1.0934	0.9870	0.9907	4.2818	1.4544	34.2647				
8.1	0.1476	0.1350	8.3067	0.9964	1.0935	0.9873	0.9910	4.3339	1.4665	35.2019				
8.2	0.1458	0.1334	8.4042	0.9965	1.0935	0.9877	0.9912	4.3861	1.4784	36.1513				
8.3	0.1441	0.1318	8.5017	0.9965	1.0935	0.9879	0.9914	4.4382	1.4903	37.1128				
8.4	0.1424	0.1302	8.5993	0.9966	1.0936	0.9882	0.9916	4.4904	1.5019	38.0864				
8.5	0.1407	0.1287	8.6970	0.9967	1.0936	0.9885	0.9918	4.5426	1.5135	39.0723				
8.6	0.1391	0.1272	8.7947	0.9968	1.0937	0.9888	0.9920	4.5948	1.5249	40.0702				
8.7	0.1375	0.1257	8.8925	0.9968	1.0937	0.9890	0.9921	4.6471	1.5362	41.0803				
8.8	0.1359	0.1243	8.9903	0.9969	1.0938	0.9893	0.9923	4.6993	1.5474	42.1025				
8.9	0.1344	0.1229	9.0882	0.9970	1.0938	0.9895	0.9925	4.7516	1.5585	43.1368				
9.0	0.1329	0.1215	9.1861	0.9971	1.0938	0.9897	0.9927	4.8039	1.5694	44.1833				
9.1	0.1315	0.1202	9.2841	0.9971	1.0939	0.9900	0.9928	4.8561	1.5802	45.2418				
9.2	0.1301	0.1189	9.3821	0.9972	1.0939	0.9902	0.9930	4.9084	1.5910	46.3123				
9.3	0.1287	0.1176	9.4801	0.9972	1.0939	0.9904	0.9931	4.9608	1.6016	47.3955				
9.4	0.1273	0.1164	9.5782	0.9973	1.0940	0.9906	0.9933	5.0131	1.6120	48.4901				

Table 4. Functions for the 1-dimensional flow of air (k=1.4), continued.

ϕ	λ	M	J	T/T0	C/C*	P/P0	P/P*	A/A*	LN(A/A*)	4FL*/D	
GENERAL				ISOENERGETIC		ISENTHROPIC					FRICTIONAL
9.5	0.1260	0.1152	9.6764	0.9974	1.0940	0.9908	0.9934	5.0654	1.6224	49.5971	
9.6	0.1247	0.1140	9.7745	0.9974	1.0940	0.9910	0.9935	5.1177	1.6327	50.7161	
9.7	0.1234	0.1128	9.8728	0.9975	1.0941	0.9911	0.9937	5.1701	1.6429	51.8473	
9.8	0.1221	0.1116	9.9710	0.9975	1.0941	0.9913	0.9938	5.2225	1.6530	52.9905	
9.9	0.1209	0.1105	10.0693	0.9976	1.0941	0.9915	0.9939	5.2748	1.6629	54.1458	
10.	0.1197	0.1094	10.1676	0.9976	1.0941	0.9917	0.9940	5.3272	1.6728	55.3132	
11.	0.1089	0.0995	11.1524	0.9980	1.0944	0.9931	0.9951	5.8514	1.7667	67.6506	
12.	0.0996	0.0912	12.1398	0.9983	1.0945	0.9942	0.9959	6.3754	1.8526	81.1925	
13.	0.0922	0.0842	13.1290	0.9986	1.0947	0.9951	0.9965	6.9018	1.9318	95.9367	
14.	0.0856	0.0772	14.1199	0.9988	1.0948	0.9957	0.9970	7.4276	2.0052	111.6814	
15.	0.0799	0.0733	15.1113	0.9989	1.0949	0.9963	0.9973	7.9536	2.0736	129.0253	
20.	0.0600	0.0548	20.0839	0.9994	1.0951	0.9979	0.9985	10.5678	2.3597	252.6997	
25.	0.0480	0.0438	25.0672	0.9996	1.0952	0.9987	0.9990	13.2248	2.5821	386.2461	
30.	0.0400	0.0365	30.0560	0.9997	1.0953	0.9991	0.9993	15.8632	2.7640	529.6243	
40.	0.0300	0.0274	40.0428	0.9999	1.0954	0.9995	0.9996	21.1423	3.0519	945.7980	
50.	0.0240	0.0219	50.0356	0.9999	1.0954	0.9997	0.9998	25.4229	3.2742		
100.	0.0120	0.0110	100.0168	1.0000	1.0954	0.9999	0.9999	52.5324	3.9671		
200.	0.0060	0.0055	200.0084	1.0000	1.0954	1.0000	1.0000	105.6582	4.4602		
500.	0.0024	0.0022	500.0034	1.0000	1.0954	1.0000	1.0000	269.4409	5.5765		
INF	0.0000	0.0000	INF	1.0000	1.0954	1.0000	1.0000	INF	INF	INF	

7. Examples of the Use of the Tables

This section contains 9 illustrative examples of the use of the present tables. After a problem has been stated, the method of solution is described. Then the given numerical values are listed, and finally the computed values are given, in the order in which they are obtained. The earlier examples are simpler than the later ones, and are explained in greater detail. For example, the methods of finding c_* and ϕ are described only in problem 1, although these quantities are used in most of the other problems also. The equations referred to in the examples will be found in Section 5 of this report.

Example 1. Finding M or λ (also finding c_* and ϕ). The stagnation temperature, pressure, flow rate, and cross-section area of a stream of air are known. Find the velocity number λ and the Mach number M of the flow.

Solution. First find c_* using eq (c) and Table 2. Then find ϕ using eq (e) and Table 3. Then enter Table 4 with ϕ to find λ , M , and any other desired properties.

Given:

$$\begin{aligned}T_o &= 550^\circ\text{R} \\P &= 6 \text{ psia} \\\dot{m} &= 0.5 \text{ lbm/sec} \\A &= 11 \text{ in}^2\end{aligned}$$

Find:

$$\begin{aligned}c_* &= 0.28289 \sqrt{T_o} \\&= 6.634 \text{ Btu}^{1/2}/\text{lbm}^{1/2} \\\phi &= c_2 \frac{\text{kPA}}{c_* \dot{m}} \\\phi &= 0.20340 \frac{\text{kPA}}{c_* \dot{m}} \\\phi &= 5.666 \\\lambda &= 0.2103 \\M &= 0.1927\end{aligned}$$

Example 2. Finding other properties when M is given. The Mach number of a flow of air is known from the angle of a weak shock wave observed in the flow. The stagnation temperature and the static pressure of the flow are known. Find the temperature, stagnation pressure, and flow per unit area.

Solution. Enter Table 4 with M to find ϕ . Use linear interpolation first but, since M lies in a gray area, check the result by 4-point interpolation. A small correction in ϕ is required to give the observed value of M exactly. Since the difference between the "linear" and "4-point" values of ϕ is small, it is safe to accept the 4-point value as correct. Using this value of ϕ , find T/T_0 and P/P_0 from Table 4; from them find T and P_0 . Note that T/T_0 and P/P_0 lie in white areas, so linear interpolation is adequate. Next solve eq (e) for \dot{m}/A , and note that all quantities required for finding \dot{m}/A are known except c_* , which can be obtained from T_0 .

Given:

$$\begin{aligned} M &= 2.6 \\ T_0 &= 560^\circ\text{R} \\ P &= 2 \text{ psia} \end{aligned}$$

Find:

$$\begin{aligned} \phi \text{ (linear)} &= 0.2748 \\ \phi \text{ (4-point)} &= 0.2747 \\ T/T_0 &= 0.4251 \\ P/P_0 &= 0.0501 \\ T &= 238.1^\circ\text{R} \\ P_0 &= 39.9 \text{ psia} \\ \frac{\dot{m}}{A} &= C_2 \frac{kP}{c_*\phi} \\ c_* &= 1059 \text{ ft/sec} \\ \frac{\dot{m}}{A} &= 0.3097 \text{ lbm/in}^2 \text{ sec} \end{aligned}$$

Example 3. Choked isentropic flow, finding \dot{m} . Air flows isentropically from a tank where its stagnation temperature and pressure are known into a converging nozzle of known throat area. The flow at the throat is sonic (choked). Find the general formula for the mass rate of flow in terms of stagnation quantities and throat area, and apply it to the given problem.

Solution. Using eq (e) and eliminating c_* with eq (c), a general expression is obtained that may be solved for \dot{m} . Writing this expression for sonic flow ($\phi = 1$)

$$\dot{m} = \frac{C_2}{C_1} \frac{k P_* A_*}{\sqrt{T_o}}$$

Since P_o is known rather than P_* , this may be written

$$\dot{m} = \frac{C_2}{C_1} k \frac{P_*}{P_o} \frac{P_o A_*}{\sqrt{T_o}}$$

This is the general formula.

To apply the general formula to a particular problem, the units of the quantities involved must be specified. Then, getting C_1 from Table 2, C_2 from Table 3, and P_*/P_o from Table 4, the given values are substituted and \dot{m} is found.

Given:

$$\begin{aligned} T_o &= 600^\circ R \\ P_o &= 20 \text{ psia} \\ A_* &= 0.1 \text{ in}^2 \end{aligned}$$

Accept:

lbf as force unit
ft/sec as velocity unit
lbm/hr as flow unit

Find:

$$\begin{aligned} C_1 &= 44.747 \\ C_2 &= 1.1583 \times 10^5 \\ P_*/P_o &= 0.5283 \\ \dot{m} &= 1914 \frac{P_o A_*}{\sqrt{T_o}} \\ &= 156.3 \text{ lbm/hr.} \end{aligned}$$

Example 4. Isentropic flow, unchoked. Air flows isentropically in a converging nozzle, but the flow at the smallest cross-section (exit) is not choked. The stagnation temperature and pressure of the flow are known; and also the flow per unit area at the exit. Find the other properties of state e, the exit state.

Solution. First find \dot{m}/A_* , for the sonic reference state of the isentropic flow. For this state $\phi = 1$ and eq (e) gives $\dot{m}/A_* = C_2 k P_* / c_*$. Finding P_* from P_0 and c_* from T_0 we calculate \dot{m}/A_* . Since \dot{m}/A_e is given, we can calculate A_e/A_* . This permits us to enter Table 4, where all the desired properties of state e may be obtained. The flow is assumed to be subsonic. Note that any value of A/A_* occurs twice in Table 4, once in the subsonic range and once in the supersonic range.

Given:

$$\begin{aligned} P_0 &= 30 \text{ psia} \\ T_0 &= 1000^\circ\text{R} \\ \dot{m}/A_e &= 0.35 \text{ lbm/in}^2 \text{ sec} \end{aligned}$$

Find:

$$\begin{aligned} C_2 &= 32.174 \\ P_* &= 15.85 \text{ psia} \\ C_1 &= 44.747 \\ c_* &= 1415 \text{ ft/sec} \\ \dot{m} &= 0.5045 \text{ lbm/sec} \\ A_* & \\ \frac{A_e}{A_*} &= 1.441 \\ \phi_e &= 2.370 \\ P_e/P_0 &= 0.8687 \\ P_e &= 26.06 \text{ psia} \\ M_e &= 0.4530 \end{aligned}$$

Example 5. Fanno-line flow. A flow of air with friction at constant area and T_0 (Fanno-line flow) has pressure P_1 at station 1 and is sonic at the channel exit. The stagnation temperature, the mass rate of flow and the cross-section area of the channel are known. Find the pressure at the channel exit and the velocity of flow at station 1 and at the exit.

Solution. In Fanno-line flow, ϕ is proportional to the pressure. We first find the constant of proportionality, calculating c_* from T_0 and evaluating the parentheses in the equation $\phi = (C_2 kA / c_* \dot{m}) P$. Then, putting $P = P_1$ we find ϕ_1 from which λ_1 may be found. To find u_1 we use eq (j). The pressure P_* at the exit is found by putting $\phi = 1$ in the equation relating ϕ and P . The velocity at the exit is simply c_* .

Given:

$$\begin{aligned} T_0 &= 530^\circ\text{R} \\ \dot{m} &= 650 \text{ lbm/hr} \\ A &= 0.25 \text{ in}^2 \\ P_1 &= 31 \text{ psia} \end{aligned}$$

Find:

$$\begin{aligned} C_1 &= 44.747 \\ c_* &= 1030 \text{ ft/sec} \\ C_2 &= 1.1583 \times 10^5 \\ C_2 kA / c_* \dot{m} &= 0.06054 \\ \phi &= 0.06054P \quad (P \text{ in psia}) \\ \phi_1 &= 1.877 \\ \lambda_1 &= 0.6009 \\ u_1 &= 619.1 \text{ ft/sec} \\ P_* &= 16.52 \text{ psia} \end{aligned}$$

Example 6. Rayleigh-line flow. Air flows in a constant-area channel without friction, but with heat addition. The velocity number and the stagnation temperature are known at station 1. Downstream, heat is added and the flow chokes at the channel exit, station 2. Find the stagnation temperature when choking occurs.

Solution. In a Rayleigh-line flow the impulse J remains constant. Since $j = kJ / c_* \dot{m}$, the quantity $c_* j$ remains constant when J is constant. Entering Table 4 with λ_1 we find j_1 . From T_{01} we calculate c_{*1} . Then $c_* j$ for the Rayleigh-line can be found. Since the exit flow is choked, j_2 is known. Equating $c_{*2} j_2$ to the known value $c_{*1} j_1$, we can find c_{*2} , and from c_{*2} we then get T_{02} .

Given:

$$\lambda_1 = 0.6$$

$$T_{o1} = 530^\circ\text{R}$$

$$\lambda_2 = 1 \text{ (choked flow)}$$

Find:

$$C_1 = 44.747$$

$$c_{*1} = 1030 \text{ ft/sec}$$

$$j_1 = 2.7200$$

$$c_{*1}j_1 = 2802 \text{ ft/sec}$$

$$j_2 = 2.400$$

$$c_{*2} = 1168 \text{ ft/sec}$$

$$T_{o2} = 681^\circ\text{R}$$

Example 7. Isothermal flow and choking. Air flows isothermally with friction in a constant-area channel. Heat exchange between the stream and its surroundings takes place by some mechanism (such as radiation) that avoids the coupling that usually exists between heat transfer and friction. The temperature, flow per unit area, and inlet (state 1) pressure are known. The downstream pressure is sufficiently low so that the flow at the channel exit (state 2) is choked. Find the stagnation temperatures and pressures of both states and the static pressure of state 2.

Solution. When T is known and T_o is not, it is easier to find M than either ϕ or λ . First calculate the stream velocity at the inlet (u_1) using the equation of continuity in the form $u = \dot{m}RT/PA$, and taking from Table 1 the value of R that will give u in the desired units. Next calculate $c = \sqrt{kRT}$ taking from Table 1 the value of R that will give c in the same units as those of u . Calculate $M_1 \equiv u_1/c$; enter Table 4 with M_1 . Find ϕ_1 , then T/T_{o1} , which gives T_{o1} ; and P_1/P_{o1} , which gives P_{o1} . To calculate state 2, make use of the fact that isothermal choking occurs when $M = 1/\sqrt{k}$. Entering Table 4 with this value of M_2 , find ϕ_2 , T/T_{o2} , and P_2/P_{o2} . From T/T_{o2} get T_{o2} . From eq (e), $P^2 = \phi c_{*} \dot{m} / C_2 k A$. Get c_{*2} from T_{o2} and C_2 from Table 3 and calculate P_2 . Finally, with P_2/P_{o2} calculate P_{o2} .

Given:

$$T = 1200^{\circ}\text{R}$$

$$\dot{m}/A = 0.25 \text{ lbm/in}^2 \text{ sec}$$

$$P_1 = 40 \text{ psia}$$

$$M_2 = 1/\sqrt{k} = 0.8452$$

Find:

$$R = 53.342 \text{ ft lbf/lbm}^{\circ}\text{R}$$

$$u_1 = 400.1 \text{ ft/sec}$$

$$R = 1716.2 \text{ ft}^2/\text{sec}^2\text{ }^{\circ}\text{R}$$

$$c = 1698 \text{ ft/sec}$$

$$M_1 = 0.2356$$

$$\phi_1 = 4.624$$

$$T/T_{01} = 0.9890$$

$$T_{01} = 1213^{\circ}\text{R}$$

$$P_1/P_{01} = 0.9621$$

$$P_{01} = 41.58 \text{ psia}$$

$$\phi_2 = 1.212$$

$$T/T_{02} = 0.8750$$

$$T_{02} = 1371^{\circ}\text{R}$$

$$P_2 = \phi_2 c_{*2} \dot{m}/C_2 kA$$

$$c_{*2} = 1657 \text{ ft/sec}$$

$$C_2 = 32.174$$

$$P_2 = 11.15 \text{ psia}$$

$$P_2/P_{02} = 0.6266$$

$$P_{02} = 17.80 \text{ psia}$$

Example 8. Friction. An air flow is carried in a circular channel of constant cross section; it is isoenergetic but subject to friction. The stagnation temperature and flow per unit area are known. The pressure is measured at station 1 and at station 2, the latter being a known number of pipe diameters downstream from the former. Find the friction factor f , assuming it to be constant between the two stations.

Solution. The friction equation (a0) must be evaluated. First calculate ϕ_1 and ϕ_2 ; enter Table 4 with these values and find $(4fL_*/D)_1$ and $(4fL_*/D)_2$. Since the friction function lies in a gray area, both linear and 4-point interpolation are used. Corresponding values are so near together that the 4-point results are probably mathematically correct. However, in view of the relatively low accuracy of most friction calculations, the values of $4fL_*/D$ are rounded off to 3 decimal places. The value of f is calculated from eq (a0).

Given:

$$T_0 = 700^\circ\text{R}$$

$$\dot{m}/A = 830 \text{ lbm/hr in}^2$$

$$P_1 = 18 \text{ psia}$$

$$P_2 = 16 \text{ psia}$$

$$\Delta(L/D) = 60$$

Find:

$$c_* = 1184 \text{ ft/sec}$$

$$\phi_1 = 2.970$$

$$\phi_2 = 2.640$$

$$(4fL_*/D)_1 = 3.0774 \text{ linear}$$

$$= 3.0770 \text{ 4-point}$$

$$= 3.077 \text{ rounded}$$

$$(4fL_*/D)_2 = 2.1649 \text{ linear}$$

$$= 2.1647 \text{ 4-point}$$

$$= 2.165 \text{ rounded}$$

$$\Delta(4fL_*/D) = 0.912$$

$$f = 0.00380$$

Example 9. Normal shock. A flow of air undergoes a normal shock. Given the upstream values of P and ϕ , find the pressure after the shock; also find the stagnation pressures before and after the shock.

Solution. We use the fact that across a normal shock $\lambda_2 = 1/\lambda_1$, eq (at). Entering Table 4 with ϕ_1 we get λ_1 . Calculating λ_2 from λ_1 we next get ϕ_2 from Table 4. Referring to eq (d) we see that all quantities in it remain constant across a normal shock except ϕ and P . Hence $\phi_2/\phi_1 = P_2/P_1$ and P_2 can be found. Then, getting P_1/P_{o1} and P_2/P_{o2} from Table 4, we calculate P_{o1} and P_{o2} .

Given:

$$P_1 = 4 \text{ psia}$$

$$\phi_1 = 0.5$$

Find:

$$\lambda_1 = 1.5$$

$$\lambda_2 = 0.6667$$

$$\phi_2 = 1.6667$$

$$P_2 = 13.33 \text{ psia}$$

$$(P/P_o)_1 = 0.1930$$

$$(P/P_o)_2 = 0.7639$$

$$P_{o1} = 20.72 \text{ psia}$$

$$P_{o2} = 17.45 \text{ psia}$$

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13. ABSTRACT New tables for the compressible flow of air have been prepared, based on 1-dimensional flow with $k = 1.4$. A new dimensionless force variable $\phi = kPa/c_m$ is introduced and it is shown that this variable is independent of the stagnation pressure of the flow. This feature makes the new tables applicable to flows of different stagnation temperatures and pressures in much the same way that conventional tables are applicable to flows of different stagnation temperatures only. The argument of the new tables is the force-variable ϕ ; the functions tabulated are $\lambda (M^*)$, M , j , T/T_0 , c/c_* , P/P_0 , ρ/ρ_0 , A/A_* , $\ln (A/A_*)$, and $4fL_*/D$. The tables are divided into white regions, in which linear interpolation introduces no significant error; and gray regions, in which linear interpolation can introduce error.			

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Supersonic flow	8,9					
Air flow	8,9					
Entropy	8,9					
Fanno-line flow	8,9					
Rayleigh-line flow	8,9					

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